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Grade 12

MATHEMATICS

Paper 2

Topic 3: Trigonometry

Practice Questions

ALL THE BEST!!

QUESTION 5

5.1 Do NOT use a calculator to answer this question. Show ALL calculations.

Prove that:

$$5.1.1 \quad \frac{\tan 480^\circ \cdot \sin 300^\circ \cdot \cos 14^\circ \cdot \sin(-135^\circ)}{\sin 104^\circ \cdot \cos 225^\circ} = \frac{3}{2} \quad (6)$$

$$5.1.2 \quad \cos 75^\circ = \frac{\sqrt{2}(\sqrt{3} - 1)}{4} \quad (4)$$

5.2 Prove that $\cos(90^\circ - 2x) \cdot \tan(180^\circ + x) + \sin^2(360^\circ - x) = 3 \sin^2 x$ (6)
[16]

QUESTION 6

6.1 6.1.1 Prove that $(\tan x - 1)(\sin 2x - 2 \cos^2 x) = 2(1 - 2 \sin x \cos x)$ (5)

6.1.2 Determine the general solution for: $\frac{\tan x - 1}{2} = -3$ correct to ONE decimal place. (5)

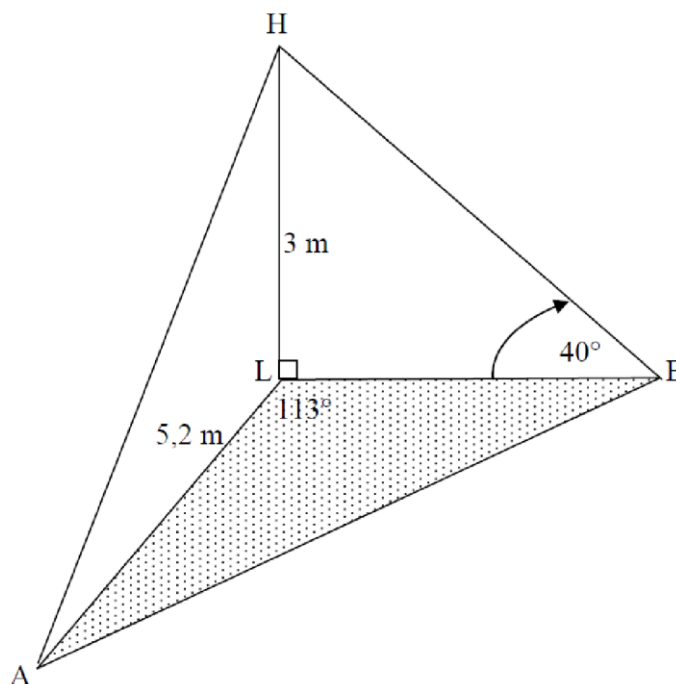
6.2 If $\cos \beta = \frac{p}{\sqrt{5}}$ where $p < 0$ and $\beta \in [180^\circ; 360^\circ]$, determine, using a diagram, an expression in terms of p for:

6.2.1 $\tan \beta$ (4)

6.2.2 $\cos 2\beta$ (3)
[17]

QUESTION 7

A, B and L are points in the same horizontal plane, HL is a vertical pole of length 3 metres, $AL = 5,2$ m, the angle $\hat{ALB} = 113^\circ$ and the angle of elevation of H from B is 40° .



- 7.1 Calculate the length of LB. (2)
 - 7.2 Hence, or otherwise, calculate the length of AB. (4)
 - 7.3 Determine the area of $\triangle ABL$. (4)
- [10]**

QUESTION 8

Consider the functions $f(x) = \cos 3x$ and $g(x) = \sin x$ for $x \in [-90^\circ; 180^\circ]$.

- 8.1 Solve for x if $f(x) = g(x)$. (8)
 - 8.2 Sketch the graphs of f and g on the system of axes on DIAGRAM SHEET 2 for $x \in [-90^\circ; 180^\circ]$. (6)
 - 8.3 Solve for x if $f(x) \leq g(x)$ where $x \in [-90^\circ; 0^\circ]$. (3)
- [17]**

QUESTION 5

Penalise 1 mark for treating as an equation in this question.

5.1.1	$\frac{\tan 480^\circ \cdot \sin 300^\circ \cdot \cos 14^\circ \cdot \sin(-135^\circ)}{\sin 104^\circ \cdot \cos 225^\circ}$ $= \frac{\tan 120^\circ \cdot (-\sin 60^\circ) \cdot \cos 14^\circ \cdot (-\sin 45^\circ)}{\sin 76^\circ \cdot (-\cos 45^\circ)}$ $= \frac{(-\tan 60^\circ) \cdot (-\sin 60^\circ) \cdot \cos 14^\circ \cdot (-\sin 45^\circ)}{\cos 14^\circ \cdot (-\cos 45^\circ)}$ $= \frac{(-\sqrt{3}) \left(-\frac{\sqrt{3}}{2}\right) \left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)}$ $= \frac{3}{2}$ OR $\frac{\tan 480^\circ \cdot \sin 300^\circ \cdot \cos 14^\circ \cdot \sin(-135^\circ)}{\sin 104^\circ \cdot \cos 225^\circ}$ $= \frac{\tan 120^\circ \cdot (-\sin 60^\circ) \cdot \cos 14^\circ \cdot (-\sin 45^\circ)}{\sin 76^\circ \cdot (-\cos 45^\circ)}$ $= \frac{(-\tan 60^\circ) \cdot (-\sin 60^\circ) \cdot \sin 76^\circ \cdot \tan 45^\circ}{\sin 76^\circ}$ $= \left(-\sqrt{3}\right) \left(-\frac{\sqrt{3}}{2}\right) 1$ $= \frac{3}{2}$	✓ $-\sin 60^\circ$ ✓ $-\sin 45^\circ$ ✓ $-\cos 45^\circ$ ✓ $-\tan 60^\circ$ ✓ $\cos 14^\circ$ or $\sin 76^\circ$ ✓ substitution Penalise 1 mark for treating as an equation in this question. (6)
5.1.2	$\cos 75^\circ$ $= \cos(45^\circ + 30^\circ)$ $= \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \sin 30^\circ$ $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ $= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$ $= \frac{\sqrt{2} \cdot \sqrt{3} - \sqrt{2}}{4}$ $= \frac{\sqrt{2}(\sqrt{3} - 1)}{4}$ OR	✓ $\cos(45^\circ + 30^\circ)$ ✓ expansion ✓ substitution ✓ simplification (4)

	$\begin{aligned} \cos 75^\circ &= \cos(45^\circ + 30^\circ) \\ &= \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\ &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3} - 1}{2\sqrt{2}} \end{aligned}$	✓ $\cos(45^\circ + 30^\circ)$ ✓ expansion ✓ substitution ✓ simplification (4)
5.2	$\begin{aligned} \cos(90^\circ - 2x) \cdot \tan(180^\circ + x) + \sin^2(360^\circ - x) \\ &= \sin 2x \cdot \tan x + \sin^2 x \\ &= 2 \sin x \cdot \cos x \cdot \frac{\sin x}{\cos x} + \sin^2 x \\ &= 2 \sin^2 x + \sin^2 x \\ &= 3 \sin^2 x \end{aligned}$	✓ $\sin 2x$ ✓ $\tan x$ ✓ $\sin^2 x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\sin 2x = 2 \sin x \cdot \cos x$ ✓ $2 \sin^2 x$ (6) If uses $\cos 2x$ instead of $\sin 2x$ and then works correctly: max 3/6 [16]

QUESTION 6

6.1.1	$ \begin{aligned} &(\tan x - 1)(\sin 2x - 2 \cos^2 x) \\ &= \left(\frac{\sin x}{\cos x} - 1 \right) (2 \sin x \cos x - 2 \cos^2 x) \\ &= \left(\frac{\sin x}{\cos x} - 1 \right) 2 \cos x (\sin x - \cos x) \\ &= 2(\sin x - \cos x)^2 \\ &= 2(\sin^2 x - 2 \sin x \cos x + \cos^2 x) \\ &= 2(1 - 2 \sin x \cos x) \end{aligned} $ OR $ \begin{aligned} &(\tan x - 1)(\sin 2x - 2 \cos^2 x) \\ &= \left(\frac{\sin x}{\cos x} - 1 \right) (2 \sin x \cos x - 2 \cos^2 x) \\ &= 2 \sin^2 x - 2 \sin x \cos x - 2 \sin x \cos x + 2 \cos^2 x \\ &= 2(\sin^2 x - 2 \sin x \cos x + \cos^2 x) \\ &= 2(1 - 2 \sin x \cos x) \end{aligned} $ OR $ \begin{aligned} &2(1 - 2 \sin x \cos x) \\ &= 2(\sin^2 x + \cos^2 x - 2 \sin x \cos x) \\ &= 2(\sin x - \cos x)^2 \\ &= 2 \cos^2 x \left(\frac{\sin x}{\cos x} - 1 \right)^2 \\ &= 2 \cos^2 x (\tan x - 1)(\tan x - 1) \\ &= (2 \cos^2 x \tan x - 2 \cos^2 x)(\tan x - 1) \\ &= (2 \sin x \cos x - 2 \cos^2 x)(\tan x - 1) \\ &= (\sin 2x - 2 \cos^2 x)(\tan x - 1) \end{aligned} $ OR $ \begin{aligned} LHS &= (\tan x - 1)(\sin 2x - \cos^2 x) \\ &= \frac{\sin x - \cos x}{\cos x} (2 \sin x \cos x - \cos^2 x) \\ &= 2(\sin x - \cos x)^2 \\ RHS &= 2(\sin^2 x + \cos^2 x - 2 \sin x \cos x) \\ &= 2(\sin x - \cos x)^2 \\ &= LHS \end{aligned} $	$ \begin{aligned} &\checkmark \frac{\sin x}{\cos x} = \tan x \\ &\checkmark \sin 2x = 2 \sin x \cos x \\ &\checkmark \text{factorisation} \\ &\checkmark \text{simplification} \\ &\checkmark \sin^2 x + \cos^2 x = 1 \end{aligned} $ (5) $ \begin{aligned} &\checkmark \frac{\sin x}{\cos x} = \tan x \\ &\checkmark \sin 2x = 2 \sin x \cos x \\ &\checkmark \text{simplification} \\ &\checkmark \text{factorisation} \\ &\checkmark \sin^2 x + \cos^2 x = 1 \end{aligned} $ $ \begin{aligned} &\checkmark \sin^2 x + \cos^2 x = 1 \\ &\checkmark \text{factorisation} \\ &\checkmark \frac{\sin x}{\cos x} = \tan x \\ &\checkmark \sin 2x = 2 \sin x \cos x \\ &\checkmark \text{simplification} \end{aligned} $ $ \begin{aligned} &\checkmark \frac{\sin x}{\cos x} = \tan x \\ &\checkmark \text{simplification} \\ &\checkmark \sin 2x = 2 \sin x \cos x \\ &\checkmark \sin^2 x + \cos^2 x = 1 \\ &\checkmark \text{factorisation} \end{aligned} $
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6.1.2	$\frac{\tan x - 1}{2} = -3$ $\tan x - 1 = -6$ $\tan x = -5$ $x = -78,7^\circ + k.180^\circ$ $k \in \mathbb{Z}$ OR $\frac{\tan x - 1}{2} = -3$ $\tan x - 1 = -6$ $\tan x = -5$ $x = 101,3^\circ + k.180^\circ$ $k \in \mathbb{Z}$ OR $\frac{\tan x - 1}{2} = -3$ $\tan x - 1 = -6$ $\tan x = -5$ $x = 101,3^\circ + k.360^\circ$ or $x = 281,3^\circ + k.360^\circ$ $k \in \mathbb{Z}$ OR If the candidate has used $\tan(x - 1) = -6$ max of 2 / 5	✓ simplification ✓ simplification ✓ $-78,7^\circ$ ✓ $+k.180^\circ$ ✓ $k \in \mathbb{Z}$ (5)
6.2.1	$\cos \beta = \frac{p}{\sqrt{5}}$ $x = p$ $r = \sqrt{5}$ $y = -\sqrt{5 - p^2}$ $\therefore \tan \beta = \frac{-\sqrt{5 - p^2}}{p}$	✓ third quadrant ✓ $y = -\sqrt{5 - p^2}$ ✓✓ answer If p is negative: 3/4 (4)

6.2.2	$\cos 2\beta = 2 \cos^2 \beta - 1$ $= 2 \left(\frac{p}{\sqrt{5}} \right)^2 - 1$ $= \frac{2p^2}{5} - 1$ OR $\cos 2\beta = 1 - 2 \sin^2 \beta$ $= 1 - 2 \left(\frac{-\sqrt{5-p^2}}{\sqrt{5}} \right)^2$ $= 1 - \frac{2(5-p^2)}{5}$ $= \frac{2p^2 - 5}{5}$ OR $\cos 2\beta = \cos^2 \beta - \sin^2 \beta$ $= \left(\frac{p}{\sqrt{5}} \right)^2 - \left(\frac{-\sqrt{5-p^2}}{\sqrt{5}} \right)^2$ $= \frac{p^2}{5} - \frac{5-p^2}{5}$ $= \frac{2p^2 - 5}{5}$	$\checkmark 2 \cos^2 \beta - 1$ $\checkmark \checkmark 2 \left(\frac{p}{\sqrt{5}} \right)^2 - 1 \text{ or}$ $\frac{2p^2}{5} - 1$ (3) $\checkmark 1 - 2 \sin^2 \beta$ $\checkmark \checkmark 1 - 2 \left(\frac{-\sqrt{5-p^2}}{\sqrt{5}} \right)^2$ $\text{or } 1 - \frac{2(5-p^2)}{5}$ $\text{or } \frac{2p^2 - 5}{5}$ (3) $\checkmark \cos^2 \beta - \sin^2 \beta$ $\checkmark \left(\frac{p}{\sqrt{5}} \right)^2$ $\checkmark \left(\frac{-\sqrt{5-p^2}}{\sqrt{5}} \right)^2$ $\text{or } \frac{p^2}{5} - \frac{5-p^2}{5}$ $\text{or } \frac{2p^2 - 5}{5}$ (3) [17]
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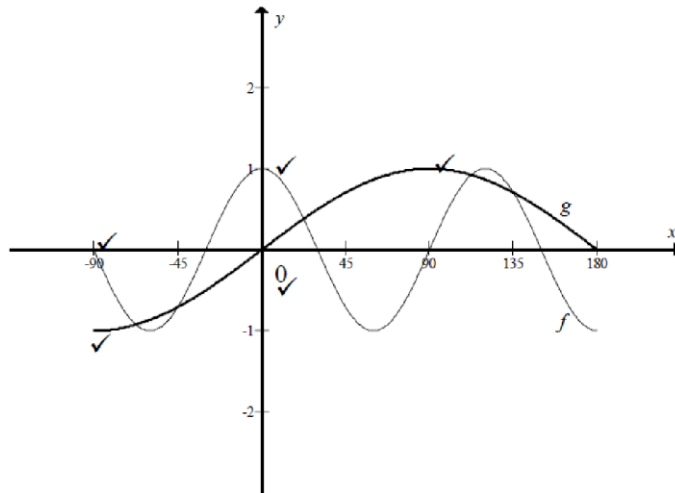
QUESTION 7

7.1	$\frac{3}{LB} = \tan 40^\circ$ $LB = \frac{3}{\tan 40^\circ}$ $LB = 3,58 \text{ m} \quad (3,5752.....)$ (3,5 m ; 3,57 m ; 3,6 m) OR $\frac{LB}{\sin 50^\circ} = \frac{3}{\sin 40^\circ}$ $LB = \frac{3 \sin 50^\circ}{\sin 40^\circ}$ $LB = 3,58 \text{ m} \quad (3,5752.....)$	✓ trig ratio ✓ answer (2) ✓ sine rule ✓ answer (2)
7.2	$AB^2 = AL^2 + BL^2 - 2 \cdot AL \cdot BL \cdot \cos 113^\circ$ $AB^2 = (5.2)^2 + (3.58)^2 - 2(5.2)(3.58) \cos 113^\circ$ $AB^2 = 54,40410138 \text{ m}^2$ $AB = 7,38 \text{ m} \quad (7,37591.....)$ Note: AB = 7,3 m or 7,4 m: accept	✓ use of cos rule ✓ substitution ✓ $AB^2 = 54,4041...m^2$ ✓ answer (4) Do not penalise if units are omitted.
7.3	$\text{Area of } \triangle ABL = \frac{1}{2} AL \cdot BL \cdot \sin \hat{A}LB$ $= \frac{1}{2} (5.2)(3.58) \sin 113^\circ$ $= 8.568059176$ $= 8,57 \text{ m}$ Note: Area = 8,5 or 8,6 : accept	✓ formula ✓ substitution ✓ ✓ answer (4) If $\cos \hat{A}LB : 0/4$ [10]

QUESTION 8

8.1	$\cos 3x = \sin x$ $\sin(90^\circ - 3x) = \sin x$ $90^\circ - 3x = x + k.360^\circ$ $90^\circ - 3x = 180^\circ - x + k.360^\circ \quad k \in \mathbb{Z}$ $-4x = -90^\circ + k.360^\circ$ or $-2x = 90^\circ + k.360^\circ$ $x = 22,5^\circ - k.90^\circ \quad k \in \mathbb{Z}$ $x = -45^\circ - k.180^\circ \quad k \in \mathbb{Z}$ $x = -67,5^\circ; 22,5^\circ; 112,5^\circ$ $x = -45^\circ; 135^\circ$ OR $\cos 3x = \cos(90^\circ - x)$ $3x = 90^\circ - x + k.360^\circ$ $3x = 360^\circ - (90^\circ - x) + k.360^\circ$ $4x = 90^\circ + k.360^\circ$ or $2x = 270^\circ + k.360^\circ$ $x = 22,5^\circ + k.90^\circ \quad k \in \mathbb{Z}$ $x = 135^\circ + k.180^\circ \quad k \in \mathbb{Z}$ $x = -67,5^\circ; 22,5^\circ; 112,5^\circ$ $x = -45^\circ; 135^\circ$ OR $\cos 3x = \cos(90^\circ - x)$ $3x = 90^\circ - x + k.360^\circ$ $3x = -90^\circ + x + k.360^\circ$ $4x = 90^\circ + k.360^\circ$ or $2x = -90^\circ + k.360^\circ$ $x = 22,5^\circ + k.90^\circ \quad k \in \mathbb{Z}$ $x = -45^\circ - k.180^\circ \quad k \in \mathbb{Z}$ $x = -67,5^\circ; 22,5^\circ; 112,5^\circ$ $x = -45^\circ; 135^\circ$	✓ equating ✓ $90^\circ - 3x = x + k.360^\circ$ ✓ $x = 22,5^\circ - k.90^\circ$ ✓ $90^\circ - 3x = 180^\circ - x + k.360^\circ$ ✓ $x = -45^\circ - k.180^\circ$ ✓✓✓ values of x (8) ✓ equating ✓ $3x = 90^\circ - x + k.360^\circ$ ✓ $x = 22,5^\circ + k.90^\circ$ ✓ $3x = 360^\circ - (90^\circ - x) + k.360^\circ$ ✓ $x = 135^\circ + k.180^\circ$ ✓✓✓ values of x (8) ✓ equating ✓ $3x = 90^\circ - x + k.360^\circ$ ✓ $x = 22,5^\circ + k.90^\circ$ ✓ $3x = -90^\circ + x + k.360^\circ$ ✓ $x = -45^\circ - k.180^\circ$ ✓✓✓ values of x (8) Note: If not all 5 values for x is given, the following applies 4 or 3 values : 2 marks 2 values : 1 mark 1 value : 0 marks
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8.2



(6)
Penalise with -1 going beyond the domain.

8.3

$$-67,5^\circ \leq x \leq -45^\circ$$

OR

$$x \in [-67,5^\circ; -45^\circ]$$

OR

From $-67,5^\circ$ up to and including -45°

✓✓ critical values
✓ notation

(3)

Note:

If $-67,5^\circ < x < -45^\circ$: 2/3

Half of the inequality: 1/3

If $x = -67,5^\circ$ or $x = -45^\circ$:
0/3

If answer is
 $22,5^\circ \leq x \leq 112,5^\circ$ then 2 / 3

If answer is $135^\circ \leq x \leq 180^\circ$
then 2 / 3

[17]

QUESTION 8

Given: $\sin \alpha = \frac{8}{17}$ where $90^\circ \leq \alpha \leq 270^\circ$

With the aid of a sketch and without the use of a calculator, calculate:

8.1 $\tan \alpha$ (3)

8.2 $\sin(90^\circ + \alpha)$ (2)

8.3 $\cos 2\alpha$ (3)
[8]

QUESTION 9

9.1 Simplify completely:

$$\sin(90^\circ - x) \cos(180^\circ - x) + \tan x \cdot \cos(-x) \sin(180^\circ + x) \quad (7)$$

9.2 Prove, without the use of a calculator, that $\frac{\sin 190^\circ \cos 225^\circ \tan 390^\circ}{\cos 100^\circ \sin 135^\circ} = -\frac{1}{\sqrt{3}}$ (7)

9.3 Determine the general solution of $\sin x + 2 \cos^2 x = 1$. (7)
[21]

QUESTION 10

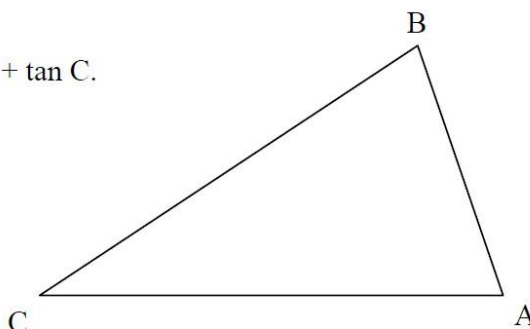
10.1 Using the expansions for $\sin(A + B)$ and $\cos(A + B)$, prove the identity of:

$$\frac{\sin(A + B)}{\cos(A + B)} = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} \quad (3)$$

10.2 If $\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)}$, prove in any $\triangle ABC$ that

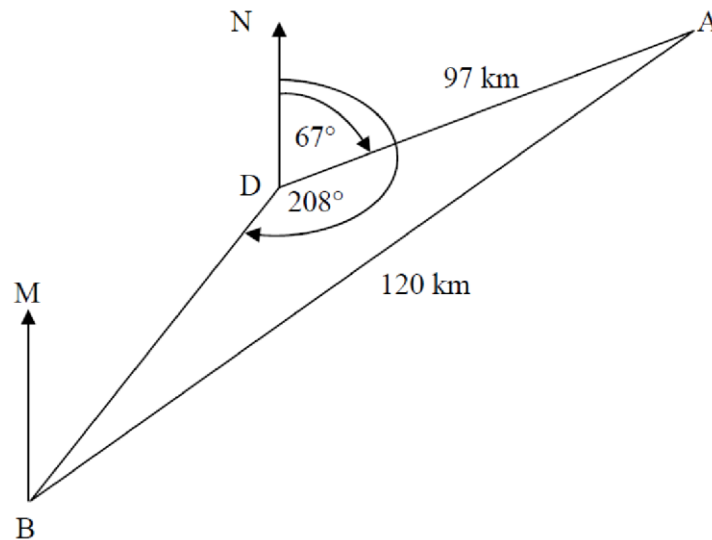
$$\tan A \cdot \tan B \cdot \tan C = \tan A + \tan B + \tan C. \quad (4)$$

[7]



QUESTION 11

- 11.1 Two ships, A and B, are 120 km apart. Ship A is at a bearing of 67° from D and 97 km away from D. DN points due north. Ship B is at a bearing of 208° from D. A diagram is provided on DIAGRAM SHEET 3.

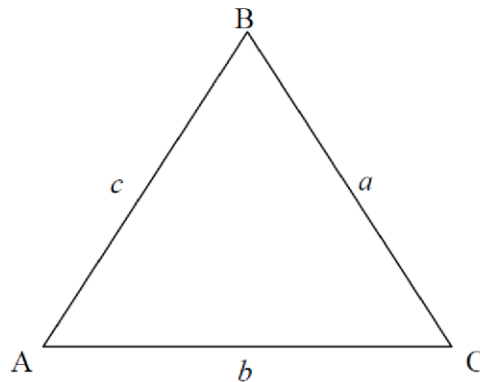


- 11.1.1 Determine the bearing of Ship A from Ship B, that is $\hat{MBA}$, when $BM \parallel DN$. (6)

- 11.1.2 If Ship B travels due north, and Ship A travels due south, then at some instant of time Ship A is due east of Ship B.

Calculate the distance between the two ships at that instant. (3)

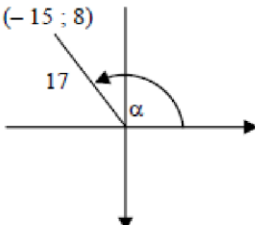
- 11.2 Triangle ABC is isosceles with $AB = BC$.



Prove that $\cos B = 1 - \frac{b^2}{2a^2}$ (4)
[13]

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 8

8.1	$\sin \alpha = \frac{8}{17}$ $\sin \alpha > 0 \therefore$ in second quadrant $y_\alpha = 8 \quad r_\alpha = 17$ $x_\alpha = -15 \quad (\text{Pythagoras})$ $\tan \alpha = -\frac{8}{15}$	 $x = -\sqrt{15}$✓ ✓ answer (3) For drawing the radius vector in the correct quadrant 1/3 Without a sketch but correct values: 3/3
8.2	$\sin(90^\circ + \alpha) = \cos \alpha$ $= -\frac{15}{17}$	✓ reduction ✓ answer (2) Answer only: full marks Cannot accept decimal values
8.3	$\cos 2\alpha = 1 - 2\sin^2 \alpha$ $= 1 - 2\left(\frac{8}{17}\right)^2$ $= \frac{161}{289}$ OR $\cos 2\alpha = 2\cos^2 \alpha - 1$ $= 2\left(\frac{-15}{17}\right)^2 - 1$ $= \frac{161}{289}$ OR $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ $= \left(\frac{-15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$ $= \frac{161}{289}$	✓ expansion ✓ substitution ✓ any further calculation or answer (3) ✓ expansion ✓ substitution ✓ any further calculation or answer (3) ✓ expansion ✓ substitution ✓ any further calculation or answer (3) [8]

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 9

NOTE: Only penalise once in the question for leaving out the x
Penalise once in this question for treating as an equation

9.1	$\sin(90^\circ - x) \cdot \cos(180^\circ - x) + \tan x \cdot \cos(-x) \cdot \sin(180^\circ + x)$ $= \cos x(-\cos x) + \tan x(\cos x)(-\sin x)$ $= -\cos^2 x - \frac{\sin x}{\cos x} \cos x \sin x$ $= -\cos^2 x - \sin^2 x$ $= -(\cos^2 x + \sin^2 x)$ $= -1$	$\checkmark \sin(90^\circ - x) = \cos x$ $\checkmark \cos(180^\circ - x) = -\cos x$ $\checkmark \cos(-x) = \cos x$ $\checkmark \sin(180^\circ + x) = -\sin x$ $\checkmark \tan x = \frac{\sin x}{\cos x}$ $\checkmark$ simplification $\checkmark$ answer (7)
9.2	$\frac{\sin 190^\circ \cos 225^\circ \tan 390^\circ}{\cos 100^\circ \sin 135^\circ}$ $= \frac{-\sin 10^\circ(-\cos 45^\circ) \tan 30^\circ}{-\sin 10^\circ \sin 45^\circ}$ $= \frac{-\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}} \quad \text{or} \quad = -\tan 30^\circ$ $= -\frac{1}{\sqrt{3}}$ If using $-\cos 80^\circ$: no penalty If the candidate stop at $= \frac{-\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{2}}} \quad 6/7$	$\checkmark \sin 190^\circ = -\sin 10^\circ$ $\checkmark \cos 225^\circ = -\cos 45^\circ$ $\checkmark \tan 390^\circ = \tan 30^\circ$ $\checkmark \cos 100^\circ = -\sin 10^\circ$ $\checkmark \sin 135^\circ = \sin 45^\circ$ or $\cos 45^\circ$ $\checkmark \checkmark$ substitution (7)
9.3	$\sin x + 2 \cos^2 x = 1$ $\sin x + 2(1 - \sin^2 x) = 1$ $-2 \sin^2 x + \sin x + 1 = 0$ $2 \sin^2 x - \sin x - 1 = 0$ $(2 \sin x + 1)(\sin x - 1) = 0$ $\sin x = 1$ $x = 90^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ Or	$\checkmark$ substitution of identity $\checkmark$ standard form $\checkmark$ factorisation $\checkmark \sin x = 1; \sin x = -\frac{1}{2}$ $\checkmark x = 90^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $\checkmark \checkmark$ answers (any two answers) (7) If $k \in \mathbb{Z}$ not included: 6/7 Also $\pm k \cdot 360^\circ; k \in \mathbb{N}_0$ or $\mathbb{Z}$

$$\sin x = -\frac{1}{2}$$

$$x = 210^\circ + k \cdot 360^\circ; k \in \mathbb{Z} \quad \text{OR} \quad x = 210^\circ + k \cdot 360^\circ$$

$$\text{or } x = 330^\circ + k \cdot 360^\circ; k \in \mathbb{Z} \quad \text{or } x = -30^\circ + k \cdot 360^\circ$$

OR

$$x = -150^\circ + k \cdot 360^\circ; k \in \mathbb{Z} \quad \text{OR} \quad x = -150^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$$

$$\text{or } x = 330^\circ + k \cdot 360^\circ \quad \text{or } x = -30^\circ + k \cdot 360^\circ$$

OR

$$\sin x + 2 \cos^2 x = 1$$

$$\sin x = 1 - 2 \cos^2 x$$

$$\sin x = -\cos 2x$$

$$\sin x = -[\sin(90^\circ - 2x)]$$

$$x = 180^\circ + (90^\circ - 2x) + k360^\circ$$

$$3x = 270^\circ + k360^\circ \quad \text{or } x = 360^\circ - (90^\circ - 2x) + k360^\circ$$

$$x = 90^\circ + k120^\circ \quad x = -270^\circ - k360^\circ$$

$$k \in \mathbb{Z}$$

OR

$$\sin x + 2 \cos^2 x = 1$$

$$\sin x = 1 - 2 \cos^2 x$$

$$\sin x = -\cos 2x$$

$$-\cos(90^\circ - x) = \cos 2x$$

$$2x = 180^\circ - (90^\circ - x) + k360^\circ \quad \text{or } 2x = 180^\circ + (90^\circ - x) + k360^\circ$$

$$x = 90^\circ + k360^\circ \quad \text{or } 3x = 270^\circ + k360^\circ$$

$$k \in \mathbb{Z} \quad x = 30^\circ + k120^\circ$$

- ✓ manipulation
- ✓ substitution of identity

- ✓ co ratios

$$\checkmark x = 180^\circ + (90^\circ - 2x) + k360^\circ$$

$$\checkmark x = 90^\circ + k120^\circ$$

$$\checkmark x = 360^\circ - (90^\circ - 2x) + k360^\circ$$

$$\checkmark x = -270^\circ - k360^\circ$$

(7)

If $k \in \mathbb{Z}$ not included: 6/7

- ✓ manipulation
- ✓ substitution of identity

- ✓ co ratios

✓

$$2x = 180^\circ - (90^\circ - x) + k360^\circ$$

$$\checkmark x = 90^\circ + k360^\circ$$

$$\checkmark 2x = 180^\circ + (90^\circ - x) + k360^\circ$$

$$\checkmark x = 30^\circ + k120^\circ$$

(7)

If $k \in \mathbb{Z}$ not included: 6/7

[20]

QUESTION 10

10.1	$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$ $= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \times \frac{\frac{1}{\cos A \cos B}}{\frac{1}{\cos A \cos B}}$ $= \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}}$ $= \frac{\tan A + \tan B}{1 - \tan A \tan B}$ OR $RHS = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A \sin B}{\cos A \cos B}} \times \frac{\cos A \cos B}{\cos A \cos B}$ $= \frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B - \sin A \sin B}$ $= \frac{\sin(A+B)}{\cos(A+B)}$ $= \tan(A+B)$ $= LHS$	✓ expansions ✓ divisions ✓ tanA and tanB (3) ✓ $\frac{\sin A}{\cos A}$ ✓ multiplication ✓ expansions (3)
10.2	$\tan C = \tan(180^\circ - (A+B))$ $\tan C = -\tan(A+B)$ $\tan C = -\left(\frac{\tan A + \tan B}{1 - \tan A \tan B}\right)$ $\tan C(1 - \tan A \tan B) = -(\tan A + \tan B)$ $\tan C - \tan A \tan B \tan C = -\tan A - \tan B$ $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ OR	✓ C ✓ $-\tan(A+B)$ ✓ substitution into formula ✓ multiplication with LCD (4) If no conclusion: 3/4

$\hat{C} = 180^\circ - (\hat{A} + \hat{B})$ (angles in a triangle) $\tan C = \tan(180^\circ - (A + B))$ $\tan C = \tan((180^\circ - A) + (-B))$ $\tan C = \frac{\tan(180^\circ - A) + \tan(-B)}{1 - \tan(180^\circ - A) \cdot \tan(-B)}$ $\tan C(1 - \tan(180^\circ - A) \cdot \tan(-B)) = \tan(180^\circ - A) + \tan(-B)$ $\tan C - \tan C \tan A \tan B = -\tan A - \tan B$ $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$	✓ C ✓ rearrange angle ✓ substitution into formula ✓ expansion (4)
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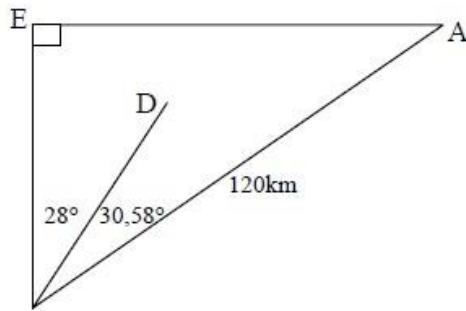
QUESTION 11

NOTE: Penalty of one for early rounding off once in this question

11.1.1	$\hat{BDA} = 208^\circ - 67^\circ$ $= 141^\circ$ $\frac{\sin \hat{DBA}}{97} = \frac{\sin 141^\circ}{120}$ $\sin \hat{DBA} = 0,5087006494...$ $\hat{DBA} = 30,58^\circ$ $\therefore$ Bearing of Ship A from Ship B $= 180^\circ - (360^\circ - 208^\circ) + 30,58^\circ$ $= 58,58^\circ$ OR $\hat{BDA} = 208^\circ - 67^\circ$ $= 141^\circ$ $\frac{\sin \hat{DBA}}{97} = \frac{\sin 141^\circ}{120}$ $\sin \hat{DBA} = 0,5087006494....$ $\hat{DBA} = 30,58^\circ$ $then\ 360^\circ - 208^\circ = \hat{NDB}$ (reflex angles) $\therefore \hat{NDB} = 152^\circ$ $but\ \hat{MBD} + \hat{NDB} = 180^\circ$ (co - interior angles/ angles around a point) $\therefore \hat{MBD} = 28^\circ$ $then\ \hat{MBA} = \hat{MBD} + \hat{DBA}$ $= 30,58^\circ + 28^\circ$ $= 58,58^\circ$	✓ $\hat{BDC} = 141^\circ$ ✓ sine rule ✓ substitution ✓ $\hat{B} = 30,58^\circ$ ✓ method or $\hat{MBD} = 28^\circ$ ✓ answer (6) ✓ $\hat{BDC} = 141^\circ$ ✓ sine rule ✓ substitution ✓ $\hat{NDB} = 152^\circ$ ✓ $\hat{MBD} = 28^\circ$ ✓ answer (6)
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11.1.2

$$\hat{B} = 30,58^\circ$$

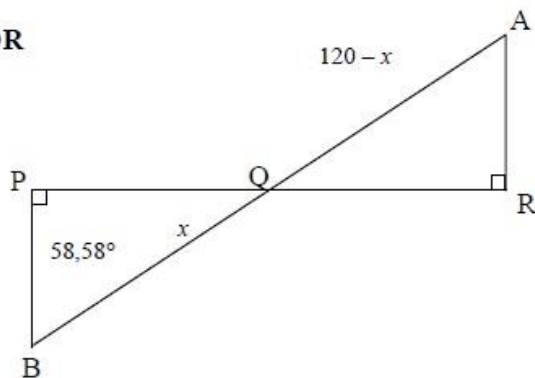


$$\frac{EA}{120} = \sin(28^\circ + 30,58^\circ)$$

$$EA = 120 \sin(28^\circ + 30,58^\circ)$$

$$EA = 102,4 \text{ km}$$

OR



Let $BQ = x$, then $AQ = 120 - x$

$$\sin 58,58^\circ = \frac{PQ}{x} \qquad \sin 58,58^\circ = \frac{QR}{120 - x}$$

$$PQ = x \cdot \sin 58,58^\circ \qquad QR = (120 - x) \sin 58,58^\circ$$

$$PQ + QR = x \cdot \sin 58,58^\circ + (120 - x) \sin 58,58^\circ$$

$$= 120 \sin 58,58^\circ$$

$$= 102,4$$

OR

$BP = AR$ (assume ships move at same speed)

✓ definition
✓ substitution

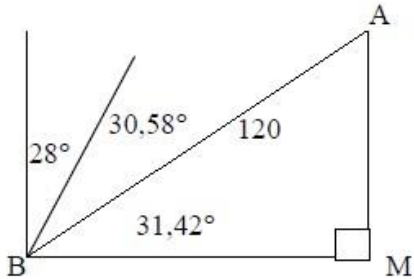
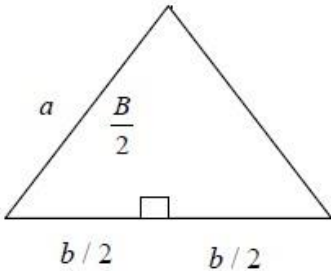
✓ answer
(3)

✓ trigonometric ratios

✓ sum

✓ answer
(3)

✓ trigonometric

	$\triangle PBQ \equiv \triangle RAQ$ (angle, angle, side) $\therefore BQ = QA = 60 \text{ km}$ $\sin 58,58^\circ = \frac{PQ}{60}$ $\therefore PQ = 60 \sin 58,58^\circ$ $= 51,20 \text{ km}$ $PR = 2PQ$ $= 102,4 \text{ km}$ OR $\frac{BM}{120} = \cos 31,42$ $BM = 120 \times \cos 31,42^\circ$ $= 102,4$ 	ratios ✓ 51,20 km ✓ answer (3) ✓ trigonometric ratios ✓ substitution ✓ answer (3)
11.2	$AB = BC = a = c$ $b^2 = a^2 + c^2 - 2ac \times \cos B$ $b^2 = a^2 + a^2 - 2a \times a \times \cos B$ $b^2 = 2a^2 - 2a^2 \cos B$ $b^2 = 2a^2(1 - \cos B)$ $\frac{b^2}{2a^2} = 1 - \cos B$ $\cos B = 1 - \frac{b^2}{2a^2}$ OR $\sin \frac{B}{2} = \frac{b}{2a}$ $\cos B = 1 - 2 \sin^2 \frac{B}{2}$ $= 1 - 2 \left(\frac{b}{2a} \right)^2$ $= 1 - \frac{b^2}{2a^2}$ 	✓ equal sides ✓ cos rule ✓ substitution ✓ simplification (4) ✓ $\sin \frac{B}{2}$ ✓ $\sin \frac{B}{2} = \frac{b}{2a}$ ✓ formula ✓ substitution (4)

OR

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

but $a = c$

$$\cos B = \frac{a^2 + a^2 - b^2}{2a.a}$$

$$= \frac{2a^2 - b^2}{2a^2}$$

$$= 1 - \frac{b^2}{2a^2}$$

[13]

✓ cos rule

✓ equal sides

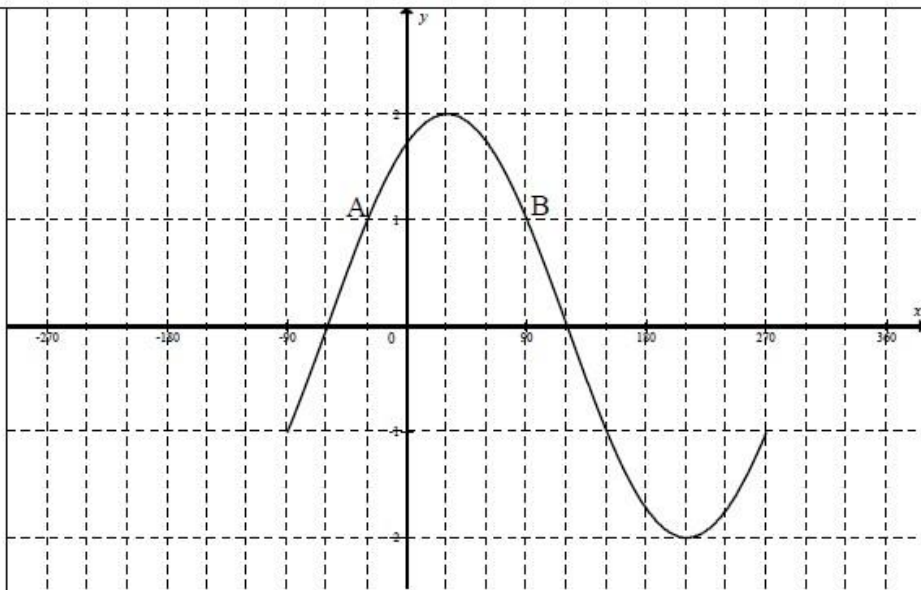
✓ substitution

✓ simplification

(4)

QUESTION 12

12.1



✓ $(120^\circ; 0)$ or $(-60^\circ; 0)$

✓ $(30^\circ; 2)$ or $(210^\circ; -2)$

(2)

QUESTION 12

12.2	$\cos(x - 30^\circ) = \frac{1}{2}$ $2 \cos(x - 30^\circ) = 1$ See points A and B on the graph Note: If drawn the line $y = \frac{1}{2}$ and put A and B on the graph: 0/2 If A and B on the x-axis: 1/2 If $A = -30^\circ$ and $B = 90^\circ$: 1/2	✓ manipulation ✓ answer (2) A and B in the correct place on the graph: full marks
12.3	$\cos(x - 30^\circ) = 0,5$ $x - 30^\circ = 60^\circ$ OR $x - 30^\circ = -60^\circ$ $x = 90^\circ$ OR $x = -30^\circ$	✓ 60° (ref angle) ✓ 90° ✓ -30° (3) Answer only: 3/3
12.4	$g'(x) = 0$ is at maximum and minimum values of graph $x = 30^\circ; 210^\circ$	✓✓ one for each x-value (2)
12.5	$x \in [-90^\circ; -60^\circ) \cup (120^\circ; 270^\circ]$ OR $-90^\circ \leq x < -60^\circ$ or $120^\circ < x \leq 270^\circ$ OR If $x < -60^\circ$ or $x > 120^\circ$ 2/3	✓ notation ✓✓ critical values (3) [12]

QUESTION 9

9.1 If $4 \tan \theta = 3$ and $180^\circ < \theta < 360^\circ$, determine with the aid of a diagram:

9.1.1 $\sin \theta + \cos \theta$ (4)

9.1.2 $\tan 2\theta$ (5)

9.2 9.2.1 Show that:
$$\frac{\cos(360^\circ - x) \tan^2 x}{\sin(x - 180^\circ) \cos(90^\circ + x)} = \frac{1}{\cos x}$$
 (5)

9.2.2 Hence, calculate without the use of a calculator, the value of:

$$\frac{\cos 330^\circ \tan^2 30^\circ}{\sin(-150^\circ) \cos 120^\circ} \quad (\text{Leave your answer in surd form.}) \quad (2)$$

[16]

QUESTION 10

10.1 If $\sin 36^\circ \cos 12^\circ = p$ and $\cos 36^\circ \sin 12^\circ = q$, determine in terms of p and q the value of:

10.1.1 $\sin 48^\circ$ (3)

10.1.2 $\sin 24^\circ$ (3)

10.1.3 $\cos 24^\circ$ (3)

10.2 Show that $\sin^2 20^\circ + \sin^2 40^\circ + \sin^2 80^\circ = \frac{3}{2}$

(HINT: $40^\circ = 60^\circ - 20^\circ$ and $80^\circ = 60^\circ + 20^\circ$) (7)

10.3 10.3.1 Prove: $\frac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x} = 1 - \cos x$ (4)

10.3.2 For which values of x is $\frac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x} = 1 - \cos x$ NOT true? (2)
[22]

QUESTION 11

Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

11.1 Calculate the points of intersection of the graphs f and g for $x \in [180^\circ; 360^\circ]$. (7)

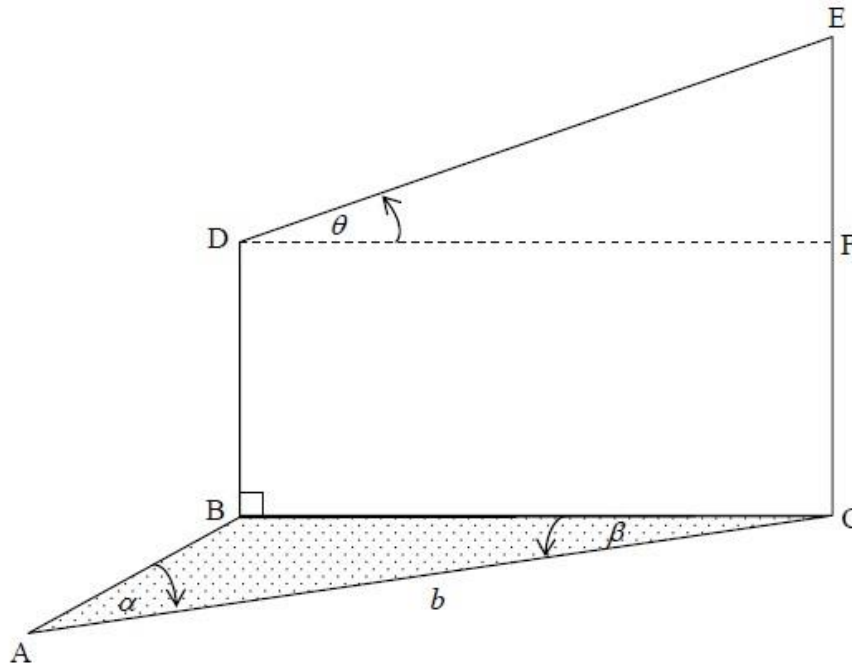
11.2 Draw sketch graphs of f and g for $x \in [180^\circ; 360^\circ]$ on the same system of axes provided on DIAGRAM SHEET 3. (4)

11.3 For which values of x will $f(x) \leq g(x)$ for $x \in [180^\circ; 360^\circ]$? (3)
[14]

QUESTION 12

In the diagram below A, B and C are three points in the same horizontal plane. D is vertically above B and E is vertically above C. The angle of elevation of E from D is θ° . F is a point on EC such that $DF \parallel BC$.

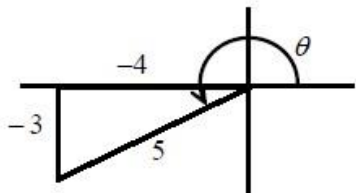
$\hat{BAC} = \alpha$, $\hat{ACB} = \beta$ and $AC = b$ metres.



12.1 Prove that $DE = \frac{b \sin \alpha}{\sin(\alpha + \beta) \cos \theta}$ (6)

12.2 Calculate DE if $b = 2\,000$ metres, $\alpha = 43^\circ$, $\beta = 36^\circ$ and $\theta = 27^\circ$. (3)
[9]

QUESTION 9

9.1.1	$\sin \theta = -\frac{3}{5}$ and $\cos \theta = -\frac{4}{5}$ $\sin \theta + \cos \theta = -\frac{7}{5}$		✓ correct quadrant and values. ✓ $\sin \theta = -\frac{3}{5}$ ✓ $\cos \theta = -\frac{4}{5}$ ✓ answer (4)
9.1.2	$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$ $= \frac{2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)}{\frac{16}{25} - \frac{9}{25}}$ $= \frac{24}{7}$ OR $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ $= \frac{2\left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2}$ $= \frac{24}{7}$	✓ $\frac{\sin 2\theta}{\cos 2\theta}$ ✓ $\sin 2\theta = 2 \sin \theta \cdot \cos \theta$ ✓ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ ✓ substitution ✓ answer (5) ✓✓ expansion ✓✓ substitution ✓ answer (5)	
9.2.1	$\frac{\cos(360^\circ - x) \cdot \tan^2 x}{\sin(x - 180^\circ) \cdot \cos(90^\circ + x)}$ $= \frac{(\cos x)(\tan^2 x)}{(-\sin x)(-\sin x)}$ $= (\cos x) \left(\frac{\sin^2 x}{\cos^2 x} \right) \left(\frac{1}{\sin^2 x} \right)$ $= \frac{1}{\cos x}$	✓ $\cos x$ ✓ $-\sin x$ ✓ $-\sin x$ ✓ $\frac{\sin^2 x}{\cos^2 x}$ ✓ answer (5)	
9.2.2	$x = 30^\circ$ $\frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$	✓ $x = 30^\circ$ ✓ answer (2)	

[16]

[16]

QUESTION 10

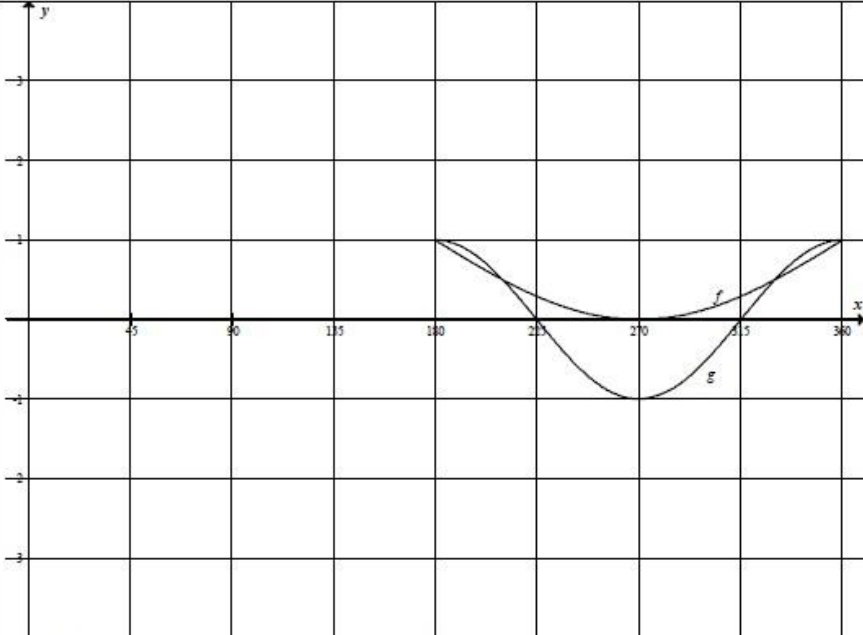
10.1.1	$\sin 48^\circ = \sin(36^\circ + 12^\circ)$ $= \sin 36^\circ \cos 12^\circ + \cos 36^\circ \sin 12^\circ$ $= p + q$	✓ writing 48° in terms of 36° and 12° ✓ expansion ✓ answer (3)
10.1.2	$\sin 24^\circ = \sin(36^\circ - 12^\circ)$ $= \sin 36^\circ \cos 12^\circ - \cos 36^\circ \sin 12^\circ$ $= p - q$ OR $\sin 24^\circ = \sin(36^\circ - 12^\circ)$ $= \sin 36^\circ \cos 12^\circ - \cos 36^\circ \sin 12^\circ$ $= p - q$	✓ writing 24° in terms of 36° and 12° ✓ expansion ✓ $\sin 24^\circ = p - q$ (3) ✓ writing 24° in terms of 36° and 12° ✓ expansion ✓ $\sin 24^\circ = p - q$ (3)
10.1.3	$\sin 48^\circ = 2 \sin 24^\circ \cos 24^\circ$ $\therefore p + q = 2(p - q) \cos 24^\circ$ $\therefore \cos 24^\circ = \frac{p + q}{2(p - q)}$ OR $\cos 48^\circ = 2 \cos^2 24^\circ - 1$ $\therefore \cos 24^\circ = \sqrt{\frac{1 + \cos 48^\circ}{2}} = \sqrt{\frac{1}{2} \left(1 + \sqrt{1 - \sin^2 48^\circ} \right)}$ $= \sqrt{\frac{1}{2} \left(1 + \sqrt{1 - (p + q)^2} \right)}$ OR $\cos^2 24^\circ = 1 - \sin^2 24^\circ$ $\cos^2 24^\circ = 1 - (p - q)^2$ $\cos 24^\circ = \sqrt{1 - (p - q)^2}$	✓ $\cos 48^\circ = 2 \cos^2 24^\circ - 1$ ✓ $\sin 48^\circ = p + q$ ✓ answer (3) ✓ $\cos 48^\circ = 2 \cos^2 24^\circ - 1$ ✓ $\sin 24^\circ = p - q$ ✓ answer (3) ✓ $\cos^2 24^\circ = 1 - \sin^2 24^\circ$ ✓ $\sin 24^\circ = p - q$ ✓ answer (3)

10.2	$\sin^2 20^\circ + \sin^2 40^\circ + \sin^2 80^\circ$ $= \sin^2 20^\circ + (\sin(60^\circ - 20^\circ))^2 + (\sin(60^\circ + 20^\circ))^2$ $= \sin^2 20^\circ + (\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)^2 + (\sin 60^\circ \cos 20^\circ + \cos 60^\circ \sin 20^\circ)^2$ $= \sin^2 20^\circ + \left(\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ\right)^2 + \left(\frac{\sqrt{3}}{2} \cos 20^\circ + \frac{1}{2} \sin 20^\circ\right)^2$ $= \sin^2 20^\circ + \frac{3}{4} \cos^2 20^\circ - \frac{\sqrt{3}}{2} \cos 20^\circ \sin 20^\circ + \frac{1}{4} \sin^2 20^\circ + \frac{3}{4} \cos^2 20^\circ$ $+ \frac{\sqrt{3}}{2} \cos 20^\circ \sin 20^\circ + \frac{1}{4} \sin^2 20^\circ$ $= \sin^2 20^\circ + \frac{3}{2} \cos^2 20^\circ + \frac{1}{2} \sin^2 20^\circ$ $= \frac{3}{2} (\sin^2 20^\circ + \cos^2 20^\circ)$ $= \frac{3}{2}$ OR Use $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ <i>LHS</i> $= \frac{3}{2} - \frac{1}{2} \{(\cos 40^\circ + \cos 80^\circ) + \cos 160^\circ\}$ $= \frac{3}{2} - \frac{1}{2} \{(\cos 60^\circ \cdot \cos 40^\circ + \sin 60^\circ \sin 40^\circ + \cos 60^\circ \cdot \cos 40^\circ - \sin 60^\circ \sin 40^\circ) + \cos 160^\circ\}$ $= \frac{3}{2} - \frac{1}{2} \{(2 \cos 60^\circ \cos 20^\circ) - \cos 20^\circ\}$ $= \frac{3}{2} - \frac{1}{2} \left\{ \left(2 \times \frac{1}{2} \cos 20^\circ \right) - \cos 20^\circ \right\}$ $= \frac{3}{2} - 0$ $= \frac{3}{2}$	✓ $40^\circ = 60^\circ - 20^\circ$ ✓ $80^\circ = 60^\circ + 20^\circ$ ✓ ✓ expansions ✓ substitution ✓ simplification ✓ factorisation (7) ✓ $40^\circ = 60^\circ - 20^\circ$ ✓ $80^\circ = 60^\circ + 20^\circ$ ✓ expansion of $\cos 40^\circ$ ✓ expansion of $\cos 60^\circ$ ✓ simplification ✓ simplification ✓ answer for bracket (7)
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10.3.1	$\frac{\sin^4 x + \sin^2 x \cos^2 x}{1 + \cos x}$ $= \frac{\sin^2 x (\sin^2 x + \cos^2 x)}{1 + \cos x}$ $= \frac{\sin^2 x}{1 + \cos x}$ $= \frac{1 - \cos^2 x}{1 + \cos x}$ $= \frac{(1 - \cos x)(1 + \cos x)}{(1 + \cos x)}$ $= 1 - \cos x$	✓ factorisation ✓ $\sin^2 x + \cos^2 x = 1$ ✓ identity ✓ factorisation (2)
10.3.2	$1 + \cos x = 0$ $\cos x = -1$ $x = 180^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ Undefined for $x = 180^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$.	✓ $1 + \cos x = 0$ ✓ $180^\circ + k \cdot 360^\circ$ (2)

QUESTION 11

11.1	$1 + \sin x = \cos 2x$ $1 + \sin x = 1 - 2 \sin^2 x$ $\sin x + 2 \sin^2 x = 0$ $\sin x (1 + 2 \sin x) = 0$ $\sin x = 0 \quad \text{or} \quad \sin x = -\frac{1}{2}$ $x = k \cdot 180 \quad \text{or} \quad x = -30^\circ + k \cdot 360 \quad k \in \mathbb{Z}$ $x = 210^\circ + k \cdot 360$ $x \in \{180^\circ; 210; 330^\circ; 360^\circ\}$ OR $1 + \sin x = \cos 2x$ $1 + \sin x = \cos^2 x - \sin^2 x$ $1 + \sin x = 1 - \sin^2 x - \sin^2 x$ $\sin x + 2 \sin^2 x = 0$ $\sin x (1 + 2 \sin x) = 0$ $\sin x = 0 \quad \text{or} \quad \sin x = -\frac{1}{2}$ $x = k \cdot 180 \quad \text{or} \quad x = -30^\circ + k \cdot 360 \quad k \in \mathbb{Z}$ $x = 210^\circ + k \cdot 360$ $x \in \{180^\circ; 210; 330^\circ; 360^\circ\}$	✓ expansion ✓ factorisation ✓ equations ✓ $x = k \cdot 180$ ✓ solution for $\sin x = -\frac{1}{2}$ ✓ ✓ answers (7) ✓ expansion ✓ factorisation ✓ equations ✓ $x = k \cdot 180$ ✓ solution for $\sin x = -\frac{1}{2}$ ✓ ✓ answers (7)
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11.2		$1 + \sin x$ ✓ max and min values ✓ shape $\cos 2x$ ✓ amplitude ✓ intercepts	(4)
11.3	$180^\circ \leq x \leq 210^\circ$ or $330^\circ \leq x \leq 360^\circ$	✓✓✓ answer	(3) [14]

QUESTION 12

12.1	$\frac{b}{\sin[180^\circ - (\alpha + \beta)]} = \frac{BC}{\sin \alpha}$ $BC \sin(\alpha + \beta) = b \sin \alpha$ $BC = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ but $BC = DF$ $\therefore DF = \frac{b \sin \alpha}{\sin(\alpha + \beta)}$ $\cos \theta = \frac{DF}{DE}$ $\therefore DE = \frac{DF}{\cos \theta}$ $\therefore DE = \frac{b \sin \alpha}{\sin(\alpha + \beta) \cos \theta}$	$\checkmark$ sine rule $\checkmark$ $\hat{A}BC = 180^\circ - (\alpha + \beta)$ $\checkmark$ $BC = \dots$ $\checkmark$ $BC = DF$ $\checkmark$ manipulation $\checkmark$ $DE = \dots$ (6)
12.2	$DE = \frac{2000 \sin 43^\circ}{\sin 79^\circ \cdot \cos 27^\circ}$ $= 1559,50 \text{ m}$	$\checkmark$ substitution numerator $\checkmark$ substitution denominator $\checkmark$ answer (3) [9]

QUESTION 9

Given: $\tan \alpha = \frac{3}{4}$; where $\alpha \in [0^\circ ; 90^\circ]$

With the use of a sketch and without the use of a calculator, calculate:

9.1 $\sin \alpha$ (3)

9.2 $\cos^2(90^\circ - \alpha) - 1$ (2)

9.3 $1 - \sin 2\alpha$ (3)
[8]

QUESTION 10

(You may NOT use a calculator to answer this question.)

10.1 Simplify completely:

$$\frac{\sin(90^\circ + \theta) + \cos(180^\circ + \theta) \sin(-\theta)}{\sin 180^\circ - \tan 135^\circ} \quad (5)$$

10.2 Prove that for any angle A:

$$\frac{4 \sin A \cos A \cos 2A \sin 15^\circ}{\sin 2A (\tan 225^\circ - 2 \sin^2 A)} = \frac{\sqrt{6} - \sqrt{2}}{2} \quad (6)$$

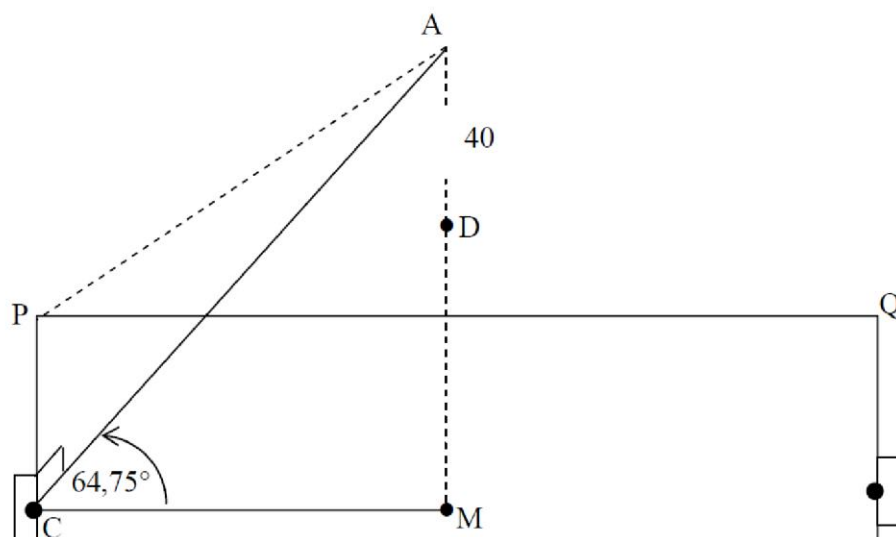
10.3 Determine the general solution of:

$$6 \cos x - 5 = \frac{4}{\cos x} \quad ; \quad \cos x \neq 0 \quad (6)$$

[17]

QUESTION 11

The angle of elevation from a point C on the ground, at the centre of the goalpost, to the highest point A of the arc, directly above the centre of the Moses Mabhida soccer stadium, is $64,75^\circ$. The soccer pitch is 100 metres long and 64 metres wide as prescribed by FIFA for world cup stadiums. Also $AC \perp PC$. In the figure below $PQ = 100$ metres and $PC = 32$ metres.



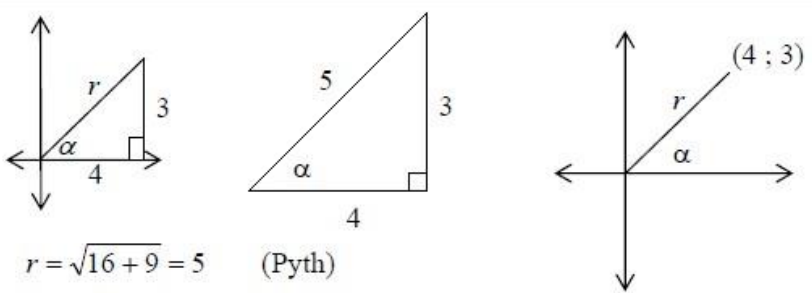
- 11.1 Determine AC. (3)
 - 11.2 Calculate $\hat{PAC}$. (3)
 - 11.3 A camera is positioned at point D, 40 metres directly below A. Calculate the distance from D to C. (4)
- [10]**

QUESTION 12

Given: $f(x) = 2 \cos x$ and $g(x) = \tan 2x$

- 12.1 Sketch the graphs of f and g on the same system of axes provided on DIAGRAM SHEET 4, for $x \in [-90^\circ ; 90^\circ]$ (6)
 - 12.2 Solve for x if $2 \cos x = \tan 2x$ and $x \in [-90^\circ ; 90^\circ]$. Show ALL working details. (8)
 - 12.3 Use the graph to solve for x : $2 \cos x \cdot \tan 2x > 0$. (4)
 - 12.4 Write down the period of $f\left(\frac{x}{2}\right)$. (2)
 - 12.5 Write down the equations of the asymptotes of $g(x - 25^\circ)$, where $x \in [-90^\circ ; 90^\circ]$. (2)
- [22]**

QUESTION 9

9.1	 $r = \sqrt{16 + 9} = 5$ (Pyth) $\sin \alpha = \frac{3}{5}$ Accept : 0,6	✓ any one of the diagram ✓ value of r ✓ answer (3)
9.2	$\begin{aligned} \cos^2(90^\circ - \alpha) - 1 \\ = \sin^2 \alpha - 1 \\ = \left(\frac{3}{5}\right)^2 - \frac{25}{25} \\ = \frac{-16}{25} \\ = -0,64 \end{aligned}$ OR $\begin{aligned} \cos^2(90^\circ - \alpha) - 1 \\ = \sin^2 \alpha - (\sin^2 \alpha + \cos^2 \alpha) \\ = -\cos^2 \alpha \\ = -\left(\frac{4}{5}\right)^2 \\ = \frac{-16}{25} \\ = -0,64 \end{aligned}$	✓ $\cos(90^\circ - \alpha) = \sin \alpha$ ✓ substitution of $\sin \alpha = \frac{3}{5}$ (2)
9.3	$\begin{aligned} 1 - \sin 2\alpha \\ = 1 - 2 \sin \alpha \cos \alpha \\ = 1 - 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) \\ = 1 - \frac{24}{25} \\ = \frac{1}{25} \end{aligned}$ OR $\begin{aligned} 1 - \sin 2\alpha \\ = \sin^2 \alpha - 2 \sin \alpha \cos \alpha + \cos^2 \alpha \\ = (\sin \alpha - \cos \alpha)^2 \\ = \left(\left(\frac{3}{5}\right) - \left(\frac{4}{5}\right)\right)^2 \\ = \left(-\frac{1}{5}\right)^2 \\ = \frac{1}{25} \end{aligned}$	✓ $\sin 2\alpha = 2 \sin \alpha \cos \alpha$ ✓ $\frac{24}{25}$ ✓ answer (3) ✓ $\sin^2 \alpha + \cos^2 \alpha = 1$ ✓ $\left(\left(\frac{3}{5}\right) - \left(\frac{4}{5}\right)\right)^2$ ✓ answer (3)

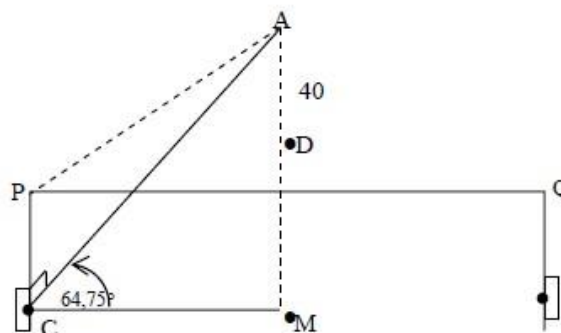
[8]

QUESTION 10

10.1	$\frac{\sin(90^\circ + \theta) + \cos(180^\circ + \theta) \sin(-\theta)}{\sin 180^\circ - \tan 135^\circ}$ $= \frac{\cos \theta + (-\cos \theta)(-\sin \theta)}{0 + 1}$ $= \cos \theta + \cos \theta \cdot \sin \theta$ $= \cos \theta(1 + \sin \theta)$	✓ $\cos \theta$ ✓ $-\cos \theta$ ✓ $-\sin \theta$ ✓ $0 + 1$ ✓ answer (5)
10.2	$\frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{\sin 2A(1 - 2 \sin^2 A)}$ $= \frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{2 \sin A \cos A(1 - 2 \sin^2 A)}$ $= \frac{2 \cos 2A \cdot \sin 15^\circ}{\cos 2A}$ $= 2 \sin 15^\circ$ $= 2 \sin(45^\circ - 30^\circ)$ $= 2[\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ]$ $= 2\left[\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right]$ $= 2\left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right]$ $= \frac{\sqrt{6} - \sqrt{2}}{2}$ OR Left Hand Side = $\frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{2 \sin A \cos A(1 - 2 \sin^2 A)}$ $= \frac{2 \cos 2A \cdot \sin 15^\circ}{\cos 2A}$ $= 2 \sin 15^\circ$ $= 2 \sin(60^\circ - 45^\circ)$ $= 2[\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ]$ $= 2\left[\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right]$ $= 2\left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right]$ $= \frac{\sqrt{6} - \sqrt{2}}{2} = RHS$	✓ $2 \sin A \cos A$ ✓ $1 - 2 \sin^2 A = \cos 2A$ ✓ $2 \sin 15^\circ$ ✓ $15 = 45 - 30$ or $15 = 60 - 45$ ✓ substitution ✓ $2\left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right]$ (6) ✓ $2 \sin A \cos A$ ✓ $1 - 2 \sin^2 A = \cos 2A$ ✓ $2 \sin 15^\circ$ ✓ $15 = 45 - 30$ or $15 = 60 - 45$ ✓ substitution ✓ $2\left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right]$ (6)

	OR $\text{Left Hand Side} = \frac{4 \sin A \cos A \cos 2A \cdot \sin 15^\circ}{2 \sin A \cos A (1 - 2 \sin^2 A)}$ $= \frac{2 \sin 2A \cos 2A \cdot \sin 15^\circ}{\sin 2A \cos 2A}$ $= 2 \sin 15^\circ$ $= 2 \sin(45^\circ - 30^\circ)$ $= 2[\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ]$ $= 2\left[\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right]$ $= 2\left[\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right]$ $= \frac{\sqrt{6} - \sqrt{2}}{2} = \text{RHS}$	
10.3	$6 \cos x - 5 = \frac{4}{\cos x}$ $6 \cos^2 x - 5 \cos x = 4$ $6 \cos^2 x - 5 \cos x - 4 = 0$ $(3 \cos x - 4)(2 \cos x + 1) = 0$ $\cos x = \frac{4}{3} \quad \text{or} \quad \cos x = -\frac{1}{2}$ no solution or $x = 120^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$ or $x = 240^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$ Alternative solution for $\cos x = -\frac{1}{2}$ $x = k \cdot 360^\circ \pm 120^\circ, k \in \mathbb{Z}$ Note: If candidate puts $\pm k \cdot 360$ then $k \in \mathbb{N}_0$	✓ standard form ✓ factors ✓ both equations ✓ $240^\circ + k \cdot 360^\circ$ ✓ $120^\circ + k \cdot 360^\circ$ ✓ $k \in \mathbb{Z}$ (6) [17]

QUESTION 11



11.1	$\cos 64,75^\circ = \frac{50}{AC}$ $AC = \frac{50}{\cos 64,75^\circ}$ $= 117,21 \text{ m}$ OR $AC = \frac{50}{\cos 64,75^\circ}$ $\therefore AC = 117,2144026 \text{ m}$ $\therefore AC = 117,21 \text{ m}$ OR $\frac{50}{\sin 25,25^\circ} = \frac{AC}{\sin 90^\circ}$ $\therefore AC = \frac{50 \sin 90^\circ}{\sin 25,25^\circ}$ $\therefore AC = 117,21 \text{ m}$	✓ substitution in ratio ✓ AC subject of the formula ✓ answer (3) ✓ substitution into ratio ✓ AC subject of the formula ✓ answer (3) ✓ substitution into sine rule ✓ AC subject of the formula ✓ answer (3)
11.2	PC is given to be $\frac{1}{2}(64) = 32 \text{ m}$ $\tan \hat{PAC} = \frac{32}{117,21}$ $\theta = 15,27^\circ \quad (15,27042173...)$ Note: If the candidate takes the unrounded answer for AC, then the answer is $15,27^\circ$ (15,26987495...)	✓ $\tan \hat{PAC}$ ✓ $\frac{32}{117,21}$ ✓ answer (3)

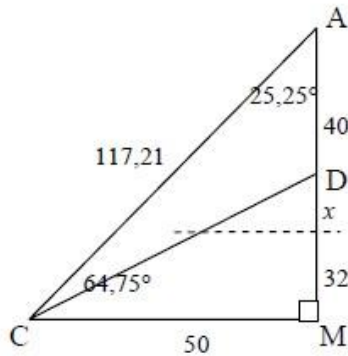
11.3

$$CD^2 = 117,21^2 + 40^2 - 2(117,21)(40) \cos 25,25$$

$$= 6857,289092$$

$$\therefore CD = 82,81 \text{ m}$$

OR



$$AM = AC \sin 64,75^\circ \text{ OR } AM = CM \tan 64,75^\circ \text{ OR } AM = AC \cos 25,25^\circ$$

$$= 106,0111876 \quad = 50 \tan 64,75^\circ \quad = 117,21 \cdot \cos 25,25^\circ$$

$$= 106,01 \quad = 106,01 \quad = 106,01$$

$$DM = 106,01 - 40$$

$$= 66,01$$

$$CD^2 = CM^2 + DM^2$$

$$= (50)^2 + (66,01)^2$$

$$= 6857,3201$$

$$CD = 82,81 \text{ metres}$$

OR

$$AM = AC \sin 64,75^\circ \text{ OR } AM = CM \tan 64,75^\circ \text{ OR } AM = AC \cos 25,25^\circ$$

$$= 106,0111876 \quad = 50 \tan 64,75^\circ \quad = 117,21 \cdot \cos 25,25^\circ$$

$$= 106,01 \quad = 106,01 \quad = 106,01$$

$$DM = 106,01 - 40$$

$$= 66,01$$

$$DC^2 = (50)^2 + (66,01)^2 - 2(50)(66,01) \cdot \cos 90^\circ$$

$$= 6857,3201$$

$$CD = 82,81 \text{ metres}$$

OR

$$\sin 64,75^\circ = \frac{40 + x + 32}{117,21}$$

$$x = 34,01$$

$$CD^2 = CM^2 + DM^2$$

$$= (50)^2 + (32 + 34,01)^2$$

$$= 6857,3201$$

$$CD = 82,81 \text{ metres}$$

Note:

If don't use the rounded off then $CD = 82,81 \text{ m}$. Accept this answer.

✓ cos rule
✓✓ substitution
✓ answer

(4)

✓ $AM = 106,01$

✓ $DM = 66,01$
✓ Pythagoras

✓ answer

(4)

✓ $AM = 106,01$

✓ $DM = 66,01$
✓ cos rule

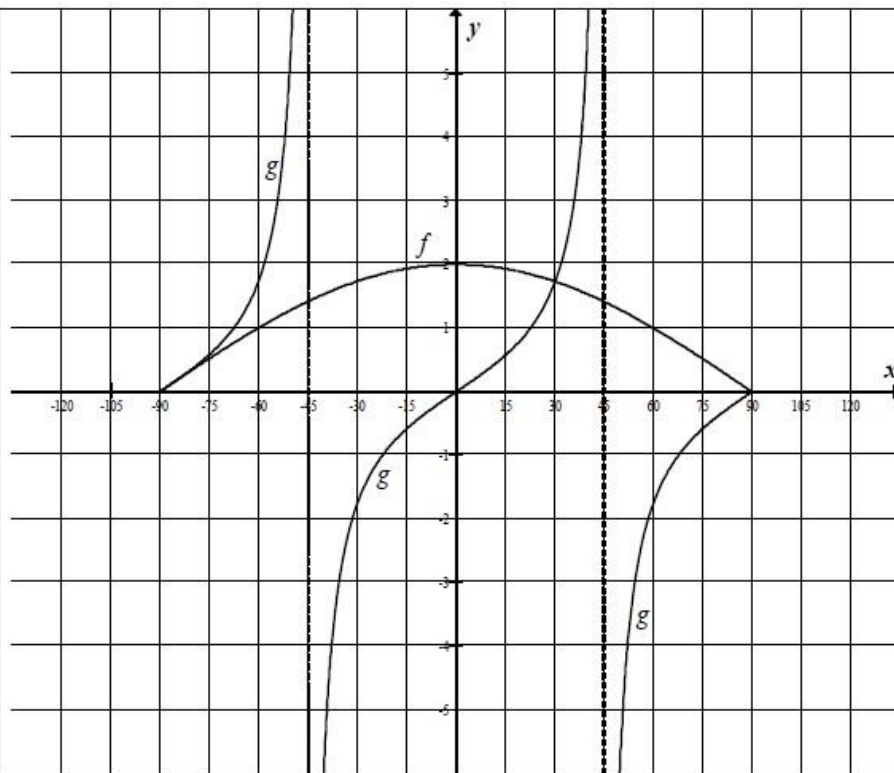
✓ answer

(4)

[10]

QUESTION 12

12.1



$\tan 2x$:
 ✓ asymptotes
 (45° & -45°)
 ✓ x -intercepts
 ✓ middle curve
 ✓ end curves
 Note: The curve
 must tend toward the
 asymptotes

$2 \cos x$:
 ✓ y -intercept
 ✓ x -intercepts

(6)

Note:
 If the candidate
 draws outside of the
 interval, NO
 PENALTY.

12.2	For these values of x, $\cos 2x \neq 0$ $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $= \frac{2 \sin x \cos x}{1 - 2 \sin^2 x}$ $1 - 2 \sin^2 x = \sin x \text{ or } \cos x = 0$ $2 \sin^2 x + \sin x - 1 = 0 \text{ or } \cos x = 0$ $(\sin x + 1)(2 \sin x - 1) = 0$ $\sin x = -1 \text{ or } \sin x = \frac{1}{2} \text{ or } \cos x = 0$ $x = \pm 90^\circ \text{ or } x = 30^\circ$ OR For these values of x, $\cos 2x \neq 0$ $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $= \frac{2 \sin x \cos x}{2 \cos^2 x - 1}$ $2 \cos x(2 \cos^2 x - 1) = 2 \sin x \cos x$ $2 \cos x(2(1 - \sin^2 x) - 1) = 2 \sin x \cos x$ $2 \cos x(1 - 2 \sin^2 x) - 2 \sin x \cos x = 0$ $2 \cos x(2 \sin^2 x + \sin x - 1) = 0$ $2 \sin^2 x + \sin x - 1 = 0 \text{ or } \cos x = 0$ $(\sin x + 1)(2 \sin x - 1) = 0$ $\sin x = -1 \text{ or } \sin x = \frac{1}{2} \text{ or } \cos x = 0$ $x = \pm 90^\circ \text{ or } x = 30^\circ$ OR $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $2 \cos x \cdot \cos 2x = \sin 2x$ $2 \cos x \cdot \cos 2x - 2 \sin x \cdot \cos x = 0$ $2 \cos x(\cos 2x - \sin x) = 0$ $\cos 2x = \sin x \text{ or } 2 \cos x = 0$ $1 - 2 \sin^2 x = \sin x$ $2 \sin^2 x + \sin x - 1 = 0$ $(\sin x + 1)(2 \sin x - 1) = 0$ $\sin x = -1 \text{ or } \sin x = \frac{1}{2} \text{ or } \cos x = 0$ $x = \pm 90^\circ \text{ or } x = 30^\circ$	✓ $\frac{\sin 2x}{\cos 2x}$ ✓ $2 \sin x \cos x$ ✓ $1 - 2 \sin^2 x$ ✓ $\cos x = 0$ ✓ factors ✓ equations ✓ $\pm 90^\circ$ ✓ 30° (8) ✓ $\frac{\sin 2x}{\cos 2x}$ ✓ $2 \sin x \cos x$ ✓ $2 \cos^2 x - 1$ ✓ $\cos x = 0$ ✓ factors ✓ equations ✓ $\pm 90^\circ$ ✓ 30° (8) ✓ $\frac{\sin 2x}{\cos 2x}$ ✓ $2 \sin x \cos x$ ✓ $1 - 2 \sin^2 x$ ✓ $\cos x = 0$ ✓ factors ✓ equations ✓ $\pm 90^\circ$ ✓ 30° (8)
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	OR $2 \cos x = \frac{\sin 2x}{\cos 2x}$ $2 \cos x \cdot \cos 2x = \sin 2x$ $2 \cos x \cdot \cos 2x - 2 \sin x \cdot \cos x = 0$ $2 \cos x (\cos 2x - \sin x) = 0$ $\cos 2x = \sin x \quad \text{or} \quad 2 \cos x = 0$ $\cos 2x = \cos(90^\circ - x) \quad x = 90^\circ$ $2x = \pm(90^\circ - x) + k \cdot 360^\circ \quad k \in \mathbb{Z}$ $3x = 90^\circ + k \cdot 360^\circ$ $x = 30^\circ + k \cdot 120^\circ$ $x = -90^\circ \quad \text{or} \quad x = 30^\circ$ or $x = -90^\circ + k \cdot 360^\circ$ $x = -90^\circ$	✓ $\frac{\sin 2x}{\cos 2x}$ ✓ $2 \sin x \cos x$ ✓ $\sin x = \cos(90^\circ - x)$ ✓ $\cos x = 0$ ✓ factors ✓ equations ✓ $\pm 90^\circ$ ✓ 30° (8)
12.3	$0^\circ < x < 45^\circ$ Or $-90^\circ < x < -45^\circ$	✓✓ critical points ✓✓ notation (4)
12.4	Period = $2(360^\circ) = 720^\circ$	✓✓ answer (2)
12.5	$x = -45^\circ + 25^\circ = -20^\circ$ $x = 45^\circ + 25^\circ = 70^\circ$ OR $2(x - 25^\circ) = -90^\circ \quad 2(x - 25^\circ) = 90^\circ$ $2x - 50^\circ = -90^\circ \quad \text{and} \quad 2x - 50^\circ = 90^\circ$ $2x = -40^\circ \quad \text{and} \quad 2x = 140^\circ$ $x = -20^\circ \quad \text{and} \quad x = 70^\circ$ Note: Answer only: full marks	✓ $x = -20^\circ$ ✓ $x = 70^\circ$ (2) [22]

QUESTION 10

10.1.1	$\cos 28^\circ = \sqrt{1 - \sin^2 28^\circ}$ $= \sqrt{1 - a^2}$	$\checkmark \sqrt{1 - \sin^2 28^\circ}$ $\checkmark$ answer (2)
10.1.2	$\cos 64^\circ$ $= \cos 2(32^\circ)$ $= 2\cos^2 32^\circ - 1$ $= 2b^2 - 1$	$\checkmark \cos 2(32^\circ)$ $\checkmark 2\cos^2 32^\circ - 1$ $\checkmark$ answer (3)
10.1.3	$\sin 4^\circ$ $= \sin(32^\circ - 28^\circ)$ $= \sin 32^\circ \cos 28^\circ - \cos 32^\circ \sin 28^\circ$ $= \sqrt{1 - b^2} \cdot \sqrt{1 - a^2} - ab$ OR $\sin 4^\circ$ $= \sin(60^\circ - 2 \times 28^\circ)$ $= \sin 60^\circ \cos(2 \times 28^\circ) - \cos 60^\circ \sin(2 \times 28^\circ)$ $= \frac{\sqrt{3}}{2} (1 - 2a^2) - \frac{1}{2} (2a) \sqrt{1 - a^2}$ $= \frac{\sqrt{3}}{2} - \sqrt{3}a^2 - a\sqrt{1 - a^2}$ OR $\sin 4^\circ$ $= \sin(2 \times 32^\circ - 60^\circ)$ $= \sin(2 \times 32^\circ) \cos 60^\circ - \cos(2 \times 32^\circ) \cdot \sin 60^\circ$ $= 2b\sqrt{1 - b^2} \cdot \frac{1}{2} - \frac{\sqrt{3}}{2} (2b^2 - 1)$ $= b\sqrt{1 - b^2} - \sqrt{3}b^2 + \frac{\sqrt{3}}{2}$ OR Using $\sin(A+B) + \sin(A-B) = 2 \cdot \sin A \cdot \cos B$ With $A = 28^\circ$ and $B = 32^\circ$ $\sin 60^\circ + \sin(-4^\circ) = 2ab$ $\sin 4^\circ = \frac{\sqrt{3}}{2} - 2ab$ OR	$\checkmark \sin(32^\circ - 28^\circ)$ $\checkmark$ expansion $\checkmark \checkmark$ answer (4)

	Using $\sin(A+B) + \sin(A-B) = 2.\sin A.\cos B$ With $A = 32^\circ$ and $B = 28^\circ$ $\sin 60^\circ + \sin(4^\circ) = 2\sqrt{1-b^2}.\sqrt{1-a^2}$ $\sin 4^\circ = 2\sqrt{1-b^2}.\sqrt{1-a^2} - \frac{\sqrt{3}}{2}$ OR Using $\sin 4^\circ = 2 \sin 2^\circ.\cos 2^\circ$ and $\sin 2^\circ = \sin(30^\circ - 28^\circ) = \frac{1}{2}(\sqrt{1-a^2} - \sqrt{3}a)$ and $\sin 2^\circ = \sin(32^\circ - 30^\circ) = \frac{1}{2}(\sqrt{3}\sqrt{1-b^2} - b)$ and $\cos 2^\circ = \cos(30^\circ - 28^\circ) = \frac{1}{2}(\sqrt{3}\sqrt{1-a^2} + a)$ and $\cos 2^\circ = \cos(32^\circ - 30^\circ) = \frac{1}{2}(\sqrt{3}b + \sqrt{1-b^2})$ then $\sin 4^\circ = \frac{1}{2}\{\sqrt{3}b\sqrt{1-a^2} - 3ab + \sqrt{1-a^2}.\sqrt{1-b^2} - \sqrt{3}a\sqrt{1-b^2}\}$ OR $\sin 4^\circ = \frac{1}{2}\{3\sqrt{1-b^2}\sqrt{1-a^2} + \sqrt{3}a\sqrt{1-b^2} - \sqrt{3}b\sqrt{1-a^2} - ab\}$	
10.2	$b\sqrt{1-a^2} - a\sqrt{1-b^2}$ $= \cos 32^\circ.\sqrt{1-\sin^2 28^\circ} - \sin 28^\circ\sqrt{1-\cos^2 32^\circ}$ $= \cos 32^\circ.\cos 28^\circ - \sin 28^\circ.\sin 32^\circ$ $= \cos(32^\circ + 28^\circ)$ $= \cos 60^\circ$ $= \frac{1}{2}$	✓ substitution ✓ $\cos 28^\circ$ ✓ $\sin 32^\circ$ ✓ compound angle formula (4)
10.3.1	$\frac{\sin 130^\circ.\tan 60^\circ}{\cos 540^\circ.\tan 230^\circ.\sin 400^\circ}$ $= \frac{\sin 50^\circ \times \tan 60^\circ}{\cos 180^\circ \times \tan 50^\circ \times \sin 40^\circ}$ $= \frac{\sin 50^\circ \times \sqrt{3}}{-1 \times \frac{\sin 50^\circ}{\cos 50^\circ} \times \cos 50^\circ}$ $= -\frac{\sqrt{3} \cos 50^\circ}{\cos 50^\circ}$ $= -\sqrt{3}$	✓ $\sin 50^\circ$ ✓ $\tan 50^\circ$ ✓ $\sin 40^\circ$ ✓ $\cos 50^\circ$ ✓ $\frac{\sin 50^\circ}{\cos 50^\circ}$ ✓ -1 ✓ answer (7)

10.3.2	$(1 - \sqrt{2} \sin 75^\circ)(1 + \sqrt{2} \sin 75^\circ)$ $= 1 - 2 \sin^2 75^\circ$ $= \cos 150^\circ$ $= \frac{-\sqrt{3}}{2}$ OR $\sin 75^\circ$ $= \sin(45^\circ + 30^\circ)$ $= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$ $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ $\sqrt{2} \sin 75^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2} = a$ $(1 - \sqrt{2} \sin 75^\circ)(1 + \sqrt{2} \sin 75^\circ)$ $= (1 - a)(1 + a)$ $= 1 - a^2$ $= 1 - \left(\frac{3}{4} + \frac{1}{4} + 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \right)$ $= -\frac{\sqrt{3}}{2}$	✓ simplification ✓ $1 - 2 \sin^2 75^\circ$ ✓ $\cos 150^\circ$ ✓ answer (4)
10.4	$\sin^2 x + \cos 2x - \cos x = 0$ $\sin^2 x + (\cos^2 x - \sin^2 x) - \cos x = 0$ $\cos^2 x - \cos x = 0$ $\cos x(\cos x - 1) = 0$ $\cos x = 0 \text{ or } \cos x = 1$ $x = \pm 90^\circ + k.360^\circ \text{ or } x = 0^\circ + k.360^\circ \quad k \in \mathbb{Z}$ $= k.360^\circ$ (i.e. $x = 90^\circ + k.180^\circ$ or $x = k.360^\circ \pm 90^\circ, k \in \mathbb{Z}$)	✓ $(\cos^2 x - \sin^2 x)$ ✓ $\cos^2 x - \cos x = 0$ ✓ factors ✓ $\cos x = 0$ or $\cos x = 1$ ✓ $90^\circ + k.360^\circ$ ✓ $k.360^\circ$ ✓ $x = -90^\circ + k.360^\circ$ (7)
10.5.1	$x = 0^\circ; 90^\circ; 180^\circ$	✓✓✓ each value (3)

10.5.2

$$\begin{aligned}\frac{\cos 2x \cdot \tan x}{\sin^2 x} &= \frac{(\cos^2 x - \sin^2 x) \cdot \frac{\sin x}{\cos x}}{\sin^2 x} \\ &= \frac{\cos^2 x - \sin^2 x}{\cos x \cdot \sin x} \\ &= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} \\ &= \frac{\cos x}{\sin x} - \tan x\end{aligned}$$

✓ $(\cos^2 x - \sin^2 x)$

✓ $\frac{\sin x}{\cos x}$

✓ answer

✓ $\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$

✓ answer

(5)

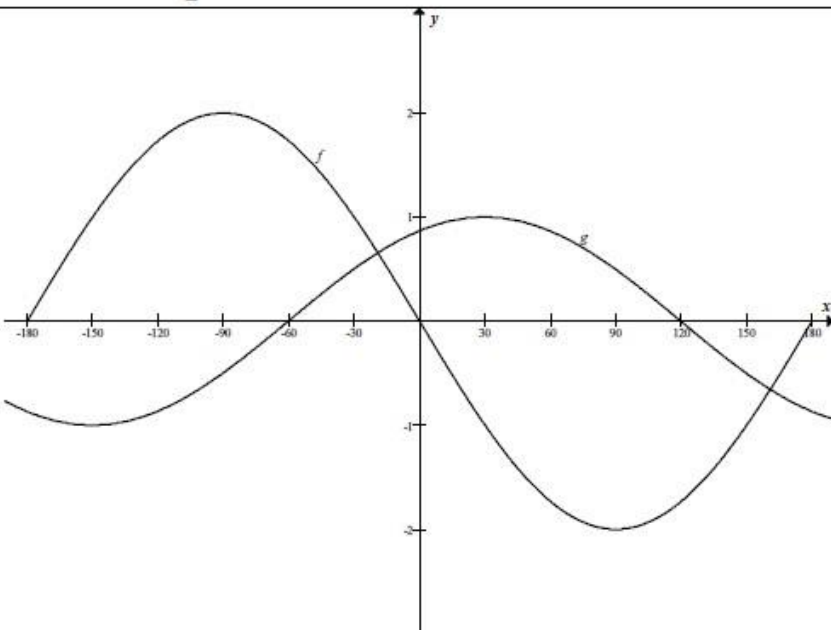
[39]

QUESTION 11

11.1	$EC^2 = DE^2 + DC^2 - 2DE \cdot DC \cos \hat{C}$ $= (7,5)^2 + (9,4)^2 - 2 \cdot (7,5)(9,4) \cos 32^\circ$ $= 25,03521844...$ $EC = 5,0 \text{ metres}$	✓ substitution into cosine rule ✓ 25,03521844... ✓ answer (3)
11.2	$\frac{\sin \hat{DCE}}{7,5} = \frac{\sin 32^\circ}{5,0}$ $\sin \hat{DCE} = \frac{7,5 \cdot \sin 32^\circ}{5,0}$ $= 0,7948788963$ $\hat{DCE} = 52,6^\circ$	✓ sin rule ✓ 0,7948788963 ✓ answer (3)
11.3	Area of $\triangle DEC$ $= \frac{1}{2} DE \cdot DC \sin \hat{D}$ $= \frac{1}{2} (7,5)(9,4) \sin 32^\circ$ $= 18,7 m^2$ OR Area of $\triangle DEC$ $= \frac{1}{2} CE \cdot DC \sin 52,6^\circ$ $= \frac{1}{2} (5,0)(9,4) \sin 52,6^\circ$ $= 18,7 m^2$	✓ substitution ✓ answer (2)
11.4	$\sin 32^\circ = \frac{EG}{7,5}$ $EG = 7,5 \cdot \sin 32^\circ$ $= 4,0$ $EF = (4 + 3,5)$ $= 7,5 \text{ metres}$ OR $EG = EC \cdot \sin 52,6^\circ$ $= (5,0) \cdot \sin 52,6^\circ$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$ OR	✓ ratio ✓ substitution ✓ answer (3) [11]

$\frac{1}{2} \cdot DC \cdot EG = \text{area } \triangle DEC$ $\frac{1}{2} (9,4) EG = 18,7$ $\therefore EG = \frac{18,7 \times 2}{9,4}$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$	
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QUESTION 12

12.1	Period = 360°	✓ answer (1)
12.2	Amplitude = $\frac{1}{2}$	✓✓ answer (2)
12.3		✓ shape ✓ x intercepts ✓ amplitude (3)
12.4	2 solutions	✓ answer (1)
12.5	$-60^\circ \leq x \leq 120^\circ$ or $x \in [-60^\circ; 120^\circ]$	✓ $-60^\circ; 120^\circ$ ✓ notation (2)
12.6	$-90^\circ < x < 30^\circ$ or $x \in (-90^\circ; 30^\circ)$	✓✓ $-90^\circ; 30^\circ$ ✓ notation (3) [12]

QUESTION 9

9.1 If $\tan A = \frac{3}{\sqrt{40}}$ and $0^\circ < \hat{A} < 90^\circ$, determine the values of the following with the aid of a sketch and without using a calculator. Leave your answers in surd form, if necessary.

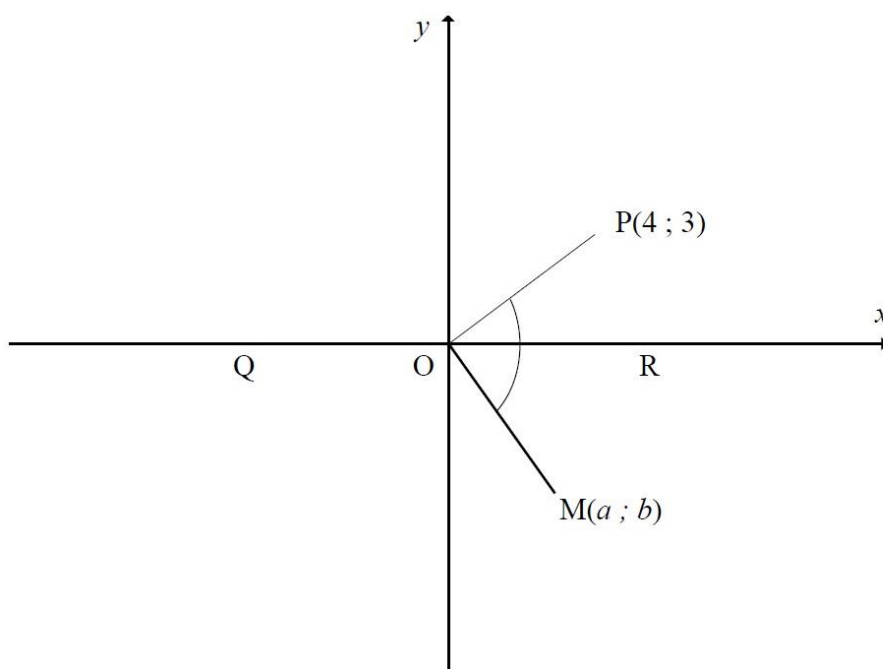
9.1.1 $\cos A$ (3)

9.1.2 $\sin (180^\circ + A)$ (2)

9.2 Without using a calculator, determine the value of the following expression:

$$\frac{\cos 100^\circ}{\sin (-10^\circ)} \times \tan^2 120^\circ$$
 (6)

9.3 P(4 ; 3) and M(a ; b) are points on a circle with the origin as centre. Q and R are x-intercepts of the circle.



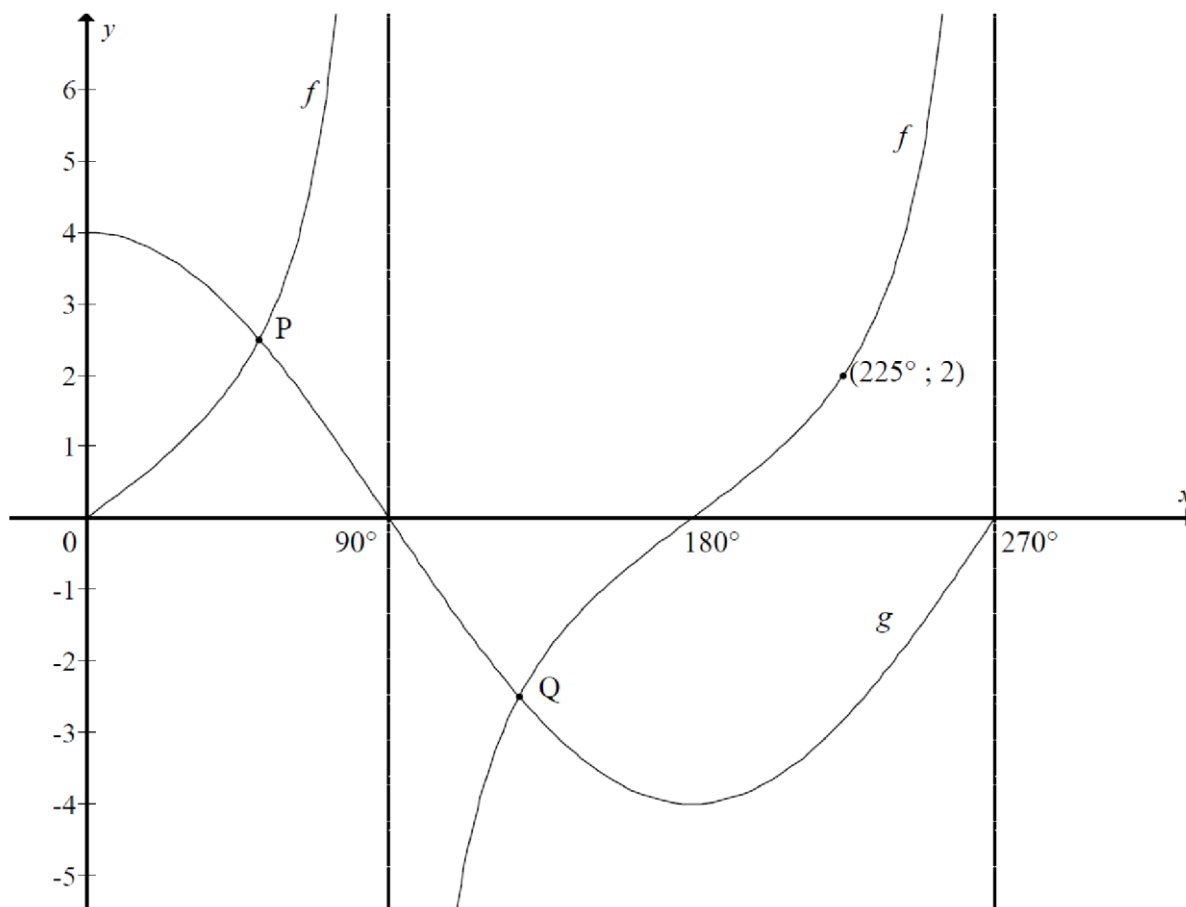
9.3.1 Write down the numerical value of $\sin \hat{R}OP$. (2)

9.3.2 Calculate the size of $\hat{Q}OP$. (2)

9.3.3 If obtuse $\hat{P}OM = 115^\circ$, calculate the value of a , the x-coordinate of M, correct to TWO decimal places. (3)
[18]

QUESTION 10

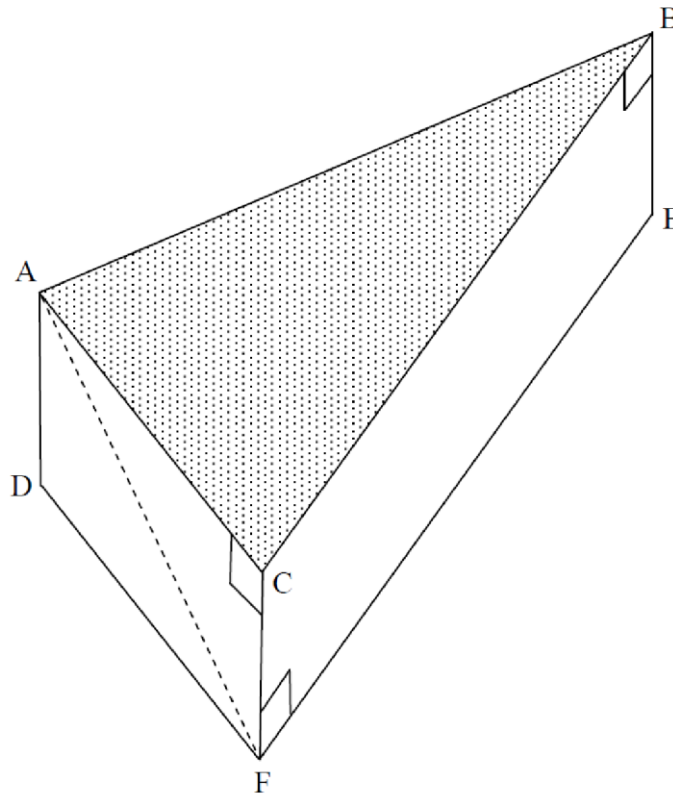
The graphs of the functions $f(x) = a \tan x$ and $g(x) = b \cos x$ for $0^\circ \leq x \leq 270^\circ$ are shown in the diagram below. The point $(225^\circ; 2)$ lies on f . The graphs intersect at points P and Q.



- 10.1 Determine the numerical values of a and b . (4)
 - 10.2 Determine the minimum value of $g(x) + 2$. (2)
 - 10.3 Determine the period of $f\left(\frac{1}{2}x\right)$. (2)
 - 10.4 Show that, if the x -coordinate of P is θ , then the x -coordinate of Q is $(180^\circ - \theta)$. (4)
- [12]**

QUESTION 11

The figure below represents a triangular right prism with $BA = BC = 5$ units, $\hat{ABC} = 50^\circ$ and $\hat{FAC} = 25^\circ$.



- 11.1 Determine the area of $\triangle ABC$. (2)
 - 11.2 Calculate the length of AC. (3)
 - 11.3 Hence, determine the height FC of the prism. (3)
- [8]

QUESTION 12

12.1 Prove that, if $\cos(\alpha - x) \neq 0$,

$$\frac{\sin(x + 450^\circ - \alpha)}{\cos(\alpha - x)} = 1. \quad (3)$$

12.2 Determine the general solution of $\cos 2x = 1 - 3 \cos x$. (7)

12.3 12.3.1 Prove that, for angles A and B ,

$$\frac{\sin A}{\sin B} - \frac{\cos A}{\cos B} = \frac{2 \sin(A - B)}{\sin 2B} \quad (4)$$

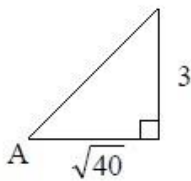
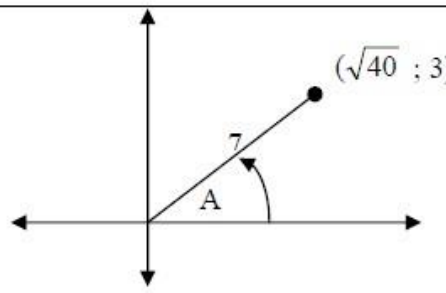
12.3.2 Hence, or otherwise, **without using a calculator**, show that:

(a)
$$\frac{\sin 5B}{\sin B} - \frac{\cos 5B}{\cos B} = 4 \cos 2B \quad (3)$$

(b)
$$\frac{1}{\sin 18^\circ} = 4 \cos 36^\circ \quad (3)$$

(c) $\sin 18^\circ$ is a solution of the cubic equation $8x^3 - 4x + 1 = 0$ (4)
[24]

QUESTION 9

9.1	9.1.1	 or  $r^2 = 40 + 9$ $r = 7$ $\cos A = \frac{\sqrt{40}}{7}$	✓ sketch ✓ $r = 7$ ✓ $\frac{\sqrt{40}}{7}$ (3)
	9.1.2	$\sin(180^\circ + A)$ $= -\sin A$ $= -\frac{3}{7}$ OR $\sin(180^\circ + A) = \sin 180^\circ \cdot \cos A + \cos 180^\circ \cdot \sin A$ $= 0 \cdot \cos A - 1 \cdot \sin A$ $= -\sin A$ $= -\frac{3}{7}$	✓ $-\sin A$ ✓ $-\frac{3}{7}$ (2) ✓ $-\sin A$ ✓ $-\frac{3}{7}$ (2)
9.2		$\frac{\cos 100^\circ \times \tan^2 120^\circ}{\sin(-10^\circ)}$ $= \frac{(-\cos 80^\circ)(-\tan 60^\circ)^2}{(-\sin 10^\circ)}$ $= \frac{(-\cos 80^\circ) \times (-\sqrt{3})^2}{(-\cos 80^\circ)}$ $= 3$ Note: Answer only: 0 marks Note: If $\frac{+\cos 80^\circ}{+\sin 10^\circ}$ (assume two negatives cancelled), no penalty	✓ $-\cos 80^\circ$ ✓ $-\tan 60^\circ$ or $\tan^2 60^\circ$ ✓ $-\sin 10^\circ$ ✓ $-\sqrt{3}$ ✓ $\sin 10^\circ = \cos 80^\circ$ ✓ 3 (6)

		OR $\frac{\cos 100^\circ \times \tan^2 120^\circ}{\sin(-10^\circ)}$ $= \frac{(-\cos 80^\circ)(-\tan 60^\circ)^2}{(-\sin 10^\circ)}$ $= \frac{(-\sin 10^\circ) \times (-\sqrt{3})^2}{(-\sin 10^\circ)}$ $= 3$ OR $\frac{\cos 100^\circ}{\sin(-10^\circ)} \times \tan^2 120^\circ$ $= \frac{\cos(90^\circ + 10^\circ)}{-\sin(10^\circ)} \times \tan^2 60^\circ$ $= \frac{-\sin 10^\circ}{-\sin 10^\circ} \times (\sqrt{3})^2$ $= 3$	$\checkmark -\cos 80^\circ$ $\checkmark -\sin 10^\circ$ $\checkmark -\tan 60^\circ$ $\checkmark -\sqrt{3}$ $\checkmark \cos 80^\circ = \sin 10^\circ$ $\checkmark 3$ (6)
9.3	9.3.1	$r = 5$ $\sin \hat{R}OP = \frac{3}{5} = 0,6$	$\checkmark 5$ $\checkmark \text{ratio}$ (2)
	9.3.2	$\hat{R}OP = 36,87^\circ$ $\hat{Q}OP = 180^\circ - 36,869....^\circ$ $\hat{Q}OP = 143,13^\circ$ Answer only: Full Marks	$\checkmark 36,869....^\circ$ $\checkmark 143,13^\circ$ (2)

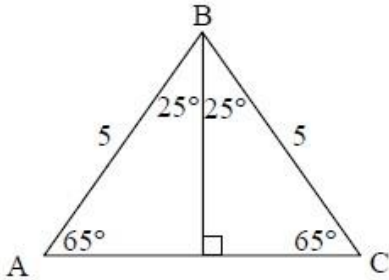
9.3.3	$x_m = x \cos \theta + y \sin \theta$ $a = 4 \cos 115^\circ + 3 \sin 115^\circ$ $a = 1,03$ Note: Penalise 1 mark for rounding incorrectly Note: If incorrect angle is used in the x- formula: 1 mark OR Rotation of 115° clockwise = 245° anticlockwise $x_m = x \cos \theta - y \sin \theta$ $a = 4 \cos 245^\circ - 3 \sin 245^\circ$ $a = 1,03$ OR $\tan \hat{POR} = \frac{3}{4}$ $\hat{POR} = 36,86...^\circ$ $\hat{MOR} = 78,13...^\circ$ $\cos \hat{MOR} = \frac{a}{5}$ $a = 5 \cos 78,13^\circ$ $a = 1,03$	✓ formula ✓ substitution of values ✓ $a = 1,03$ (3) ✓ formula ✓ substitution of values ✓ $a = 1,03$ (3) ✓ $36,86^\circ$ ✓ cos ratio ✓ $a = 1,03$ (3) [18]
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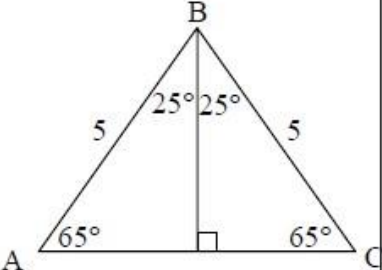
QUESTION 10

10.1	$f(225^\circ) = 2$ $\therefore a \tan 225^\circ = 2 \quad \therefore a = 2$ $g(0) = 4$ $\therefore b \cos 0^\circ = 4 \quad \therefore b = 4$ Answer only: Full marks	✓ substitution ✓ $a = 2$ ✓ substitution ✓ $b = 4$ (4)
10.2	Minimum value of $g(x) + 2 = -4 + 2 = -2$ Answer only: Full marks	✓ -4 ✓ -2 (2)
10.3	Period = $\frac{180^\circ}{\frac{1}{2}} = 360^\circ$ Answer only: Full marks	✓ $\frac{180^\circ}{\frac{1}{2}}$ ✓ 360° (2)

10.4	At P $f(\theta) = g(\theta)$ $2 \tan \theta = 4 \cos \theta$ for $180^\circ - \theta$: $2 \tan (180^\circ - \theta) = -2 \tan \theta$ and $4 \cos (180^\circ - \theta) = -4 \cos \theta$ $2 \tan \theta = 4 \cos \theta$ at P $\therefore -2 \tan \theta = -4 \cos \theta$ $\therefore 2 \tan (180^\circ - \theta) = 4 \cos (180^\circ - \theta)$ at Q OR $2 \tan \theta = 4 \cos \theta$ $\frac{\sin \theta}{\cos \theta} = 2 \cos \theta$ $\sin \theta = 2 \cos^2 \theta$ $= 2(1 - \sin^2 \theta)$ $2 \sin^2 \theta + \sin \theta - 2 = 0$ $\sin \theta = \frac{-1 \pm \sqrt{1 - 4(2)(-2)}}{4}$ $\sin \theta = 0,78077...$ $\theta = 51,33^\circ$ or $128,67^\circ$ $\therefore$ the x - coordinate of Q is $180^\circ - x_p$	$\checkmark 2 \tan \theta = 4 \cos \theta$ $\checkmark 2 \tan (180^\circ - \theta) = -2 \tan \theta$ $\checkmark 4 \cos (180^\circ - \theta) = -4 \cos \theta$ $\checkmark 2 \tan (180^\circ - \theta) = 4 \cos (180^\circ - \theta)$ (4) $\checkmark$ equation $\checkmark \sin \theta = 0,78077...$ $\checkmark 51,33^\circ$ $\checkmark 128,67^\circ$ (4) [12]
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QUESTION 11

11.1	Area $\triangle ABC = \frac{1}{2} AB \cdot BC \cdot \sin 50^\circ$ $= \frac{1}{2} (5)(5) \sin 50^\circ$ $= 9,58 \text{ units}^2$ OR Area of $\triangle ABC$ $= \frac{1}{2} (2)(5) \sin 25^\circ (5 \cos 25^\circ)$ $= 9,58 \text{ units}^2$  OR Area of $\triangle ABC$ $= \left[\frac{1}{2} (5 \cos 65^\circ) (5 \sin 65^\circ) \right] (2)$ $= 9,58 \text{ units}^2$	$\checkmark$ substitution into correct formula $\checkmark$ answer (2) $\checkmark$ base and height in terms of 5 and 25° $\checkmark$ answer (2) $\checkmark$ base and height in terms of 5 and 65° $\checkmark$ answer (2)
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11.2	$AC^2 = 5^2 + 5^2 - 2(5)(5) \cos 50^\circ$ $AC^2 = 17,86061952$ $AC = 4,23 \text{ units}$ OR $\hat{A} = \hat{C} = 65^\circ$ (angles opposite equal sides) $\frac{\sin 65^\circ}{5} = \frac{\sin 50^\circ}{AC}$ $AC = \frac{5 \sin 50^\circ}{\sin 65^\circ}$ $= 4,23 \text{ units}$ OR $\sin 25^\circ = \frac{\frac{1}{2}(AC)}{5}$ $AC = 2(5) \sin 25^\circ$ $= 4,23 \text{ units}$  OR $\cos 65^\circ = \frac{\frac{1}{2}(AC)}{5}$ $AC = 2(5) \cos 65^\circ$ $AC = 4,23 \text{ units}$	✓ use of cosine rule ✓ substitution ✓ answer (3) ✓ use of sine rule ✓ substitution ✓ answer (3) ✓ sketch/diagram ✓ $\sin 25^\circ = \frac{\frac{1}{2} AC}{5}$ ✓ answer (3) ✓ sketch/diagram ✓ $\cos 65^\circ = \frac{\frac{1}{2}(AC)}{5}$ ✓ answer (3)
11.3	$\tan 25^\circ = \frac{CF}{AC}$ $\therefore CF = 4,23 \times \tan 25^\circ$ $\therefore CF = 1,97 \text{ units}$ OR $\frac{FC}{\sin 25^\circ} = \frac{4,23}{\sin 65^\circ}$ $FC = \frac{4,23 \sin 25^\circ}{\sin 65^\circ}$ $= 1,97 \text{ units}$	✓ ratio ✓ CF as subject ✓ answer (3) ✓ sine rule ✓ FC as subject ✓ answer (3)

[8]

QUESTION 12

12.1	$LHS = \frac{\sin(360^\circ + 90^\circ + x - \alpha)}{\cos(\alpha - x)}$ $= \frac{\sin(90^\circ + x - \alpha)}{\cos(\alpha - x)}$ $= \frac{\cos(x - \alpha)}{\cos(\alpha - x)}$ $= \frac{\cos(\alpha - x)}{\cos(\alpha - x)}$ $= 1$ OR $LHS = \frac{\sin[90^\circ - (\alpha - x)]}{\cos(\alpha - x)}$ $= \frac{\cos(\alpha - x)}{\cos(\alpha - x)}$ $= 1$ $= RHS$	✓ subtracting 360° ✓ $\cos(x - \alpha)$ ✓ $\cos(\alpha - x)$ (3) ✓ subtracting 360° ✓ writing as $90^\circ - (\alpha - x)$ ✓ $\cos(\alpha - x)$ (3)
12.2	$\cos 2x = 1 - 3 \cos x$ $2 \cos^2 x - 1 = 1 - 3 \cos x$ $2 \cos^2 x + 3 \cos x - 2 = 0$ $(2 \cos x - 1)(\cos x + 2) = 0$ $\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = -2$ n/a $x = 60^\circ + k.360^\circ ; k \in \mathbb{Z} \quad \text{or} \quad x = 300^\circ + k.360^\circ ; k \in \mathbb{Z}$ OR $x = \pm 60^\circ + k.360^\circ ; k \in \mathbb{Z}$	✓ $\cos 2x = 2 \cos^2 x - 1$ ✓ factorisation ✓ $\cos x = \frac{1}{2}$ ✓ 60° ✓ 300° ✓ $+ k.360^\circ$ ✓ $k \in \mathbb{Z}$ (7)
12.3.1	LHS: $\frac{\sin A \cos B - \cos A \sin B}{\sin B \cos B}$ $= \frac{\sin(A - B)}{\sin B \cos B}$ RHS = $\frac{2 \sin(A - B)}{2 \sin B \cos B}$ $= \frac{\sin(A - B)}{\sin B \cos B}$ $= LHS$	✓ writing as single fraction ✓ comp. angle expansion ✓ comp. angle expansion ✓ simplification (4)

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12.3.2(a)	$A = 5B$ $\frac{\sin 5B}{\sin B} - \frac{\cos 5B}{\cos B} = \frac{2 \sin(5B - B)}{\sin 2B}$ $= \frac{2 \sin 4B}{\sin 2B}$ $= \frac{4 \sin 2B \cos 2B}{\sin 2B}$ $= 4 \cos 2B$ OR $\frac{\sin 5B}{\sin B} - \frac{\cos 5B}{\cos B}$ $= \frac{\sin 5B \cos B - \cos 5B \sin B}{\sin B \cos B}$ $= \frac{\sin(5B - B)}{\sin B \cos B}$ $= \frac{\sin 4B}{\sin 2B}$ $= \frac{1}{2} (2) \sin 2B \cos 2B$ $= \frac{2 \sin 2B \cos 2B}{2}$ $= \sin 2B \cos 2B$ $= 4 \cos 2B$	✓ recognising $A = 5B$ ✓ substituting $A = 5B$ ✓ $\sin 4B$ $= 2 \sin 2B \cos 2B$ (3) ✓ writing as single fraction ✓ $\sin 4B$ $= 2 \sin 2B \cos 2B$ ✓ compound angle in denominator (3)
12.3.2(b)	$B = 18^\circ$ $\frac{\sin 90^\circ}{\sin 18^\circ} - \frac{\cos 90^\circ}{\cos 18^\circ} = 4 \cos 2(18^\circ)$ $\therefore \frac{1}{\sin 18^\circ} - 0 = 4 \cos 36^\circ$ $\therefore \frac{1}{\sin 18^\circ} = 4 \cos 36^\circ$	✓ recognising $B = 18^\circ$ ✓ substituting $B = 18^\circ$ ✓ simplify (3)
12.3.2(c)	Let $\sin 18^\circ = a$ $\frac{1}{\sin 18^\circ} = 4 \cos 36^\circ$ $\frac{1}{\sin 18^\circ} = 4(1 - 2 \sin^2 18^\circ)$ $\therefore \frac{1}{a} = 4(1 - 2a^2)$ $\therefore 1 = 4a - 8a^3$ $\therefore 8a^3 - 4a + 1 = 0$ Hence $\sin 18^\circ$ is a solution of $\therefore 8x^3 - 4x + 1 = 0$ OR	✓ $\sin 18^\circ = a$ ✓ $\cos 36^\circ$ $= 1 - 2 \sin^2 18^\circ$ ✓ substitution of a ✓ simplification (4)

$\frac{1}{\sin 18^\circ} = 4 \cos 36^\circ$ $\frac{1}{\sin 18^\circ} = 4(1 - 2 \sin^2 18^\circ)$ $\frac{1}{\sin 18^\circ} = 4 - 8 \sin^2 18^\circ$ $8(\sin 18^\circ)^3 - 4(\sin 18^\circ) + 1 = 0$ Hence $\sin 18^\circ$ is a solution of $\therefore 8x^3 - 4x + 1 = 0$ Note: substituting $x = \sin 18^\circ$ into $8x^3 - 4x + 1$ using a calculator showing equal to 0: 0 marks	✓ $\cos 36^\circ$ $= 1 - 2 \sin^2 18^\circ$ ✓ simplification ✓ equation i.t.o $\sin 18^\circ$ ✓ replacing $\sin 18^\circ = x$ (4) [24]
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QUESTION 8

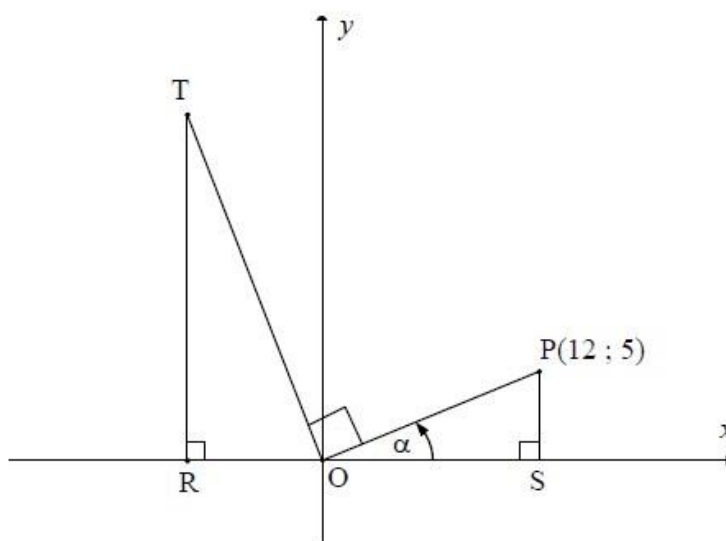
If $Q'(-2 ; -3)$ is the image of Q after rotation of 135° in an anticlockwise direction about the origin, calculate the coordinates of Q . (Leave your answer in surd form.)

[5]

QUESTION 9

9.1 Answer this question without using a calculator.

In the diagram, P is the point $(12 ; 5)$. $OT \perp OP$. PS and TR are perpendicular to the x -axis. $\hat{POS} = \alpha$ and $OR = 7,5$ units.



Determine:

9.1.1 $\cos \alpha$ (2)

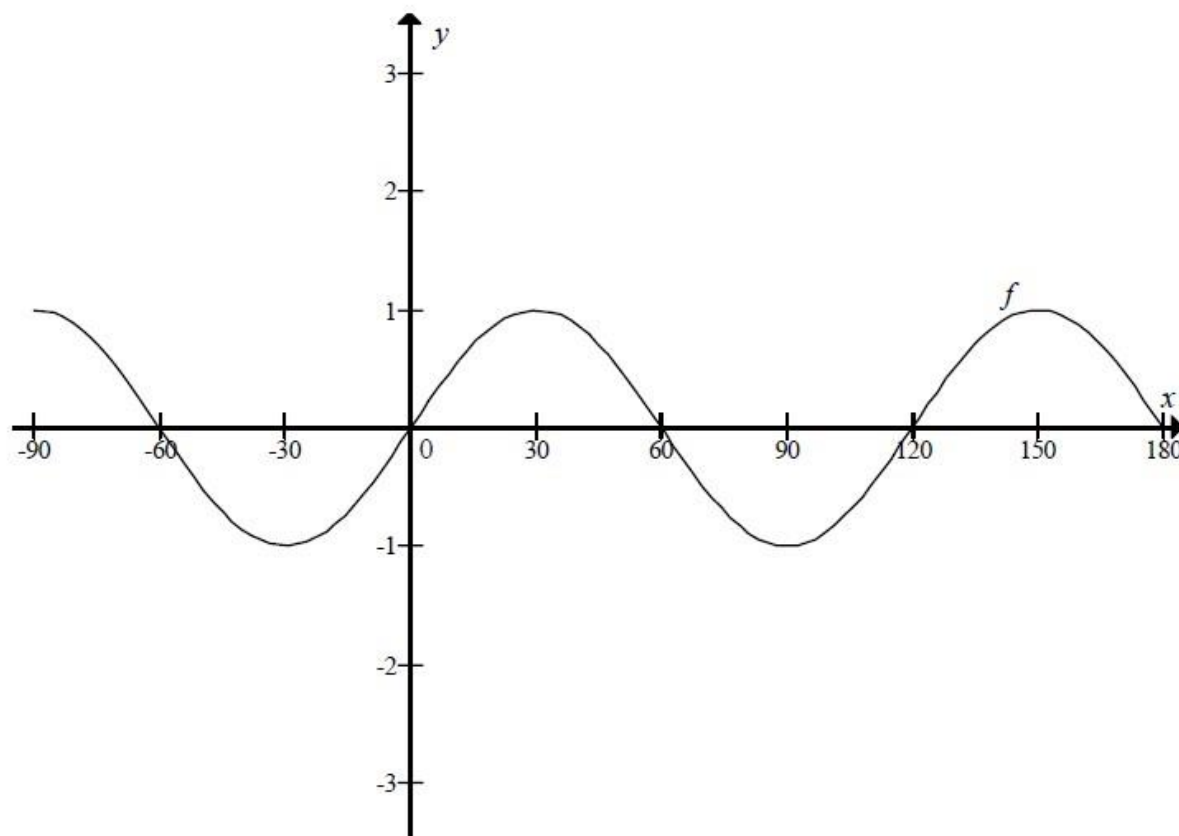
9.1.2 $\hat{TOR}$, in terms of α (2)

9.1.3 The length of OT (4)

9.2 Show that $\frac{\sin(90^\circ + x) \cdot \cos x \cdot \tan(-x)}{\cos(180^\circ + x)} = \sin x$. (4)
[12]

QUESTION 10

The graph of $f(x) = \sin 3x$ is drawn below for $x \in [-90^\circ; 180^\circ]$.



- 10.1 Write down the period of f . (1)
- 10.2 Write down the solutions for $\sin 3x = -1$ on the interval $[-90^\circ; 180^\circ]$. (2)
- 10.3 Give the maximum value of h if $h(x) = f(x) - 1$. (2)
- 10.4 Draw the graph of $g(x) = 3 \cos x$ for $x \in [-90^\circ; 180^\circ]$ on the grid on DIAGRAM SHEET 4. (3)
- 10.5 Use the graphs to determine how many solutions there are to the equation $\frac{\sin 3x}{3} - \cos x = 0$ on the interval $[-90^\circ; 180^\circ]$. (2)
- 10.6 Use the graphs to solve: $f(x) \cdot g(x) < 0$. (4)

[14]

QUESTION 11

11.1 If $\sin 61^\circ = \sqrt{p}$, determine the following in terms of p :

11.1.1 $\sin 241^\circ$ (2)

11.1.2 $\cos 61^\circ$ (2)

11.1.3 $\cos 122^\circ$ (3)

11.1.4 $\cos 73^\circ \cos 15^\circ + \sin 73^\circ \sin 15^\circ$ (3)

11.2 11.2.1 Prove the identity:

$$\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2 \tan 2x \quad (6)$$

11.2.2 Determine a value of x in the interval $[0^\circ ; 180^\circ]$ for which the identity is not valid. (2)

11.3 11.3.1 Given: $\sin x = \cos 2x - 1$. Show that $2 \sin^2 x + \sin x = 0$. (1)

11.3.2 Determine the general solution of the equation: $\sin x = \cos 2x - 1$. (6)

11.4 Determine the value of:

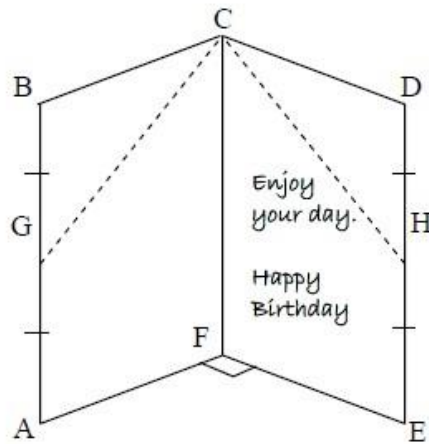
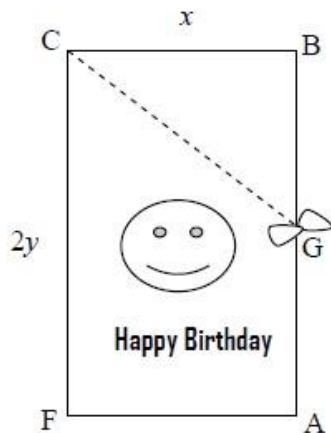
$$\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \tan 4^\circ \times \dots \times \tan 87^\circ \times \tan 88^\circ \times \tan 89^\circ. \quad (4)$$

[29]

QUESTION 12

A rectangular birthday card is tied with a ribbon at the midpoints, G and H, of the longer sides. The card is opened to read the message inside and then placed on a table in such a way that the angle $\widehat{AFE}$ between the front cover and the back cover of the card is 90° . The points G and H are joined by straight lines to the point C inside the card, as shown in the sketch.

Let the shorter side of the card, $BC = x$, and the longer side, $CF = 2y$.



Prove that $\cos \widehat{GCH} = \frac{y^2}{x^2 + y^2}$.

[8]

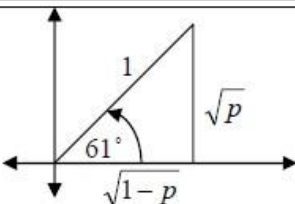
QUESTION 9

9.1.1	$r = 13$ $\cos \alpha = \frac{12}{13}$	$\checkmark 13$ $\checkmark \frac{12}{13}$ (2)
9.1.2	$\hat{TOR} = 180^\circ - (90^\circ + \alpha)$ $= 90^\circ - \alpha$	$\checkmark 180^\circ - (90^\circ + \alpha)$ $\checkmark 90^\circ - \alpha$ (2)
9.1.3	$\cos \hat{TOR} = \frac{TR}{OT}$ $\cos(90^\circ - \alpha) = \frac{7,5}{OT}$ $OT = \frac{7,5}{\cos(90^\circ - \alpha)}$ $OT = \frac{7,5}{\sin \alpha}$ $OT = \frac{7,5}{\frac{5}{13}}$ $OT = 19,5$ OR $\sin(\hat{RTO}) = \frac{7,5}{OT}$ $\therefore OT = \frac{7,5}{\sin \alpha}$ $OT = \frac{7,5}{\frac{5}{13}}$ $OT = 19,5$	$\checkmark$ $\cos(90^\circ - \alpha) = \frac{7,5}{OQ}$ $\checkmark \frac{7,5}{\sin \alpha}$ $\checkmark \frac{5}{13}$ $\checkmark 19,5$ (4) $\checkmark$ $\sin(\hat{RQO}) = \frac{7,5}{OQ}$ $\checkmark \frac{7,5}{\sin \alpha}$ $\checkmark \frac{5}{13}$ $\checkmark 19,5$ (4)
9.2	$LHS = \frac{\cos x \cdot \cos x(-\tan x)}{-\cos x}$ $= \cos x \cdot \frac{\sin x}{\cos x}$ $= \sin x$ $= RHS$	$\checkmark \cos x$ $\checkmark -\tan x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark \text{answer}$ (4) [12]

QUESTION 10

10.1	Period = 120°	✓ 120° (1)
10.2	$\sin 3x = -1$ $x = -30^\circ$ or $x = 90^\circ$	✓ -30° ✓ 90° (2)
10.3	Maximum value of $f(x)$ is 1 $\therefore$ Maximum value of $h(x)$ is 0	✓ max of $f(x)$ ✓ answer (2)
10.4		✓ $-90^\circ; 90^\circ$ ✓ $(0^\circ; 3)$ ✓ $(180^\circ; -3)$ (3)
10.5	$\frac{\sin 3x}{3} - \cos x = 0$ $\sin 3x - 3 \cos x = 0$ $\therefore \sin 3x = 3 \cos x$ There are 2 solutions where graphs f and g are equal	✓ $\sin 3x = 3 \cos x$ ✓ answer (2)
10.6	$f(x) \cdot g(x) < 0$ $x \in (-60^\circ; 0^\circ)$ or $(60^\circ; 90^\circ)$ or $(120^\circ; 180^\circ)$ OR $-60^\circ < x < 0^\circ$ or $60^\circ < x < 90^\circ$ or $120^\circ < x < 180^\circ$	✓✓✓ for each interval ✓ correct brackets or correct symbols (4) [14]

QUESTION 11

11.1.1	$\sin 61^\circ = \sqrt{p}$ $\sin 241^\circ = \sin (180^\circ + 61^\circ)$ $= -\sin 61^\circ$ $= -\sqrt{p}$		$\checkmark -\sin 61^\circ$ $\checkmark$ answer (2)
11.1.2	$\cos 61^\circ = \sqrt{1 - \sin^2 61^\circ}$ $= \sqrt{1 - p}$		$\checkmark$ identity $\checkmark$ answer (2)
11.1.3	$\cos 122^\circ = \cos 2(61^\circ)$ $= 2 \cos^2 61^\circ - 1$ $= 2(\sqrt{1 - p})^2 - 1$ $= 2(1 - p) - 1$ $= 2 - 2p - 1$ $= 1 - 2p$		$\checkmark$ double angle $\checkmark$ expansion $\checkmark$ answer (3)
11.1.4	$\cos 73^\circ \cos 15^\circ + \sin 73^\circ \sin 15^\circ$ $= \cos(73^\circ - 15^\circ)$ $= \cos 58^\circ = (\cos 180^\circ - 122^\circ)$ $= -(\cos 122^\circ)$ $= -(1 - 2p)$ $= 2p - 1$		$\checkmark \cos(73^\circ - 15^\circ)$ $\checkmark -(\cos 122^\circ)$ $\checkmark$ answer (3)
11.2.1	$\text{LHS} = \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{\cos^2 x + 2 \cos x \sin x + \sin^2 x - (\cos^2 x - 2 \sin x \cos x + \sin^2 x)}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{4 \cos x \sin x}{\cos^2 x - \sin^2 x}$ $= \frac{2 \sin 2x}{\cos 2x}$ $= 2 \tan x$ $= \text{RHS}$		$\checkmark$ $\frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ $\checkmark$ numerator $\checkmark 4 \cos x \sin x$ $\checkmark \cos^2 x - \sin^2 x$ $\checkmark 2 \sin 2x$ $\checkmark \cos 2x$ (6)
11.2.2	$\cos x = \sin x \quad \text{or} \quad \cos x = -\sin x$ $x = 45^\circ \quad \quad \quad x = 135^\circ$		$\checkmark \checkmark$ for answer (2)
11.3.1	$\sin x = \cos 2x - 1$ $\sin x = 1 - 2 \sin^2 x - 1$ $\sin x = -2 \sin^2 x$ $2 \sin^2 x + \sin x = 0$		$\checkmark 1 - 2 \sin^2 x$ (1)

11.3.2	$\sin x = \cos 2x - 1$ $2 \sin^2 x + \sin x = 0$ $\sin x (2 \sin x + 1) = 0$ $\sin x = 0 \text{ or } \sin x = -\frac{1}{2}$ $\therefore x = 0^\circ + 180^\circ k; k \in \mathbb{Z} \text{ or } x = \{210^\circ \text{ or } 330^\circ\} + 360^\circ k; k \in \mathbb{Z}$ OR $x = n.180^\circ$ $x = n.360^\circ - 30^\circ$ $x = (2n+1).180^\circ + 30^\circ, n \in \mathbb{Z}$	$\checkmark \sin x (2 \sin x + 1) = 0$ $\checkmark \sin x = 0 \text{ or } \sin x = -\frac{1}{2}$ $\checkmark 0^\circ + 180^\circ k$ $\checkmark 210^\circ$ $\checkmark 330^\circ$ $\checkmark + 360^\circ k; k \in \mathbb{Z}$ (6)
11.4	$\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \tan 4^\circ \times \dots \times \tan 87^\circ \times \tan 88^\circ \times \tan 89^\circ$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\sin 88^\circ}{\cos 88^\circ}\right) \left(\frac{\sin 89^\circ}{\cos 89^\circ}\right)$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\sin(90^\circ - 2^\circ)}{\cos(90^\circ - 2^\circ)}\right) \left(\frac{\sin(90^\circ - 1^\circ)}{\cos(90^\circ - 1^\circ)}\right)$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\cos 2^\circ}{\sin 2^\circ}\right) \left(\frac{\cos 1^\circ}{\sin 1^\circ}\right)$ $= \tan 45^\circ$ $= 1$ OR $\tan 89^\circ = \cot 1^\circ \quad \tan 88^\circ = \cot 2^\circ \dots$ $\therefore \text{product is } (\tan 1^\circ \cdot \cot 1^\circ)(\tan 2^\circ \cdot \cot 2^\circ) \dots (\tan 44^\circ \cdot \cot 44^\circ) \cdot \tan 45^\circ$ $= 1 \times 1 \times 1 \times \dots \times 1 = 1$	$\checkmark \text{ identity}$ $\checkmark \text{ co-ratios}$ $\checkmark \text{ simplification}$ $\checkmark \text{ for answer}$ (4) $\checkmark \text{ identity}$ $\checkmark \text{ co-ratios}$ $\checkmark \text{ simplification}$ $\checkmark \text{ for answer}$ (4) [29]

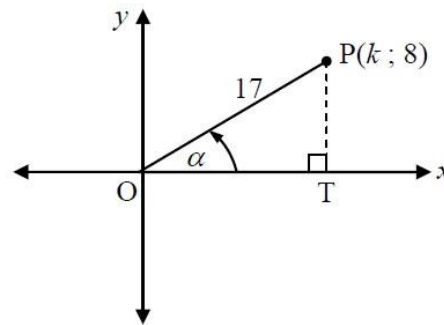
QUESTION 12

12	In ΔCBG and ΔCDH: $CG^2 = x^2 + y^2$ Pythagoras $CH^2 = x^2 + y^2$ Pythagoras In ΔFAE $AE^2 = x^2 + x^2$ $= 2x^2$ $= GH^2$ In ΔCGH $GH^2 = CG^2 + CH^2 - 2 CG \cdot CH \cdot \cos GCH$ $\cos G\hat{C}H = \frac{CG^2 + CH^2 - GH^2}{2CG \cdot CH}$ $\cos G\hat{C}H = \frac{x^2 + y^2 + x^2 + y^2 - 2x^2}{2\sqrt{x^2 + y^2} \cdot \sqrt{x^2 + y^2}}$ $\cos G\hat{C}H = \frac{2y^2}{2(x^2 + y^2)}$ $\cos G\hat{C}H = \frac{y^2}{x^2 + y^2}$	✓ CG^2 ✓ CH^2 ✓ AE^2 ✓ $AE^2 = GH^2$ ✓ use of cos rule ✓ manipulation of formula ✓ substitution ✓ $\cos G\hat{C}H = \frac{2y^2}{2(x^2 + y^2)}$ (8) [8]
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QUESTION 8

Answer this question **WITHOUT** using a calculator.

- 8.1 The point $P(k; 8)$ lies in the first quadrant such that $OP = 17$ units and $\hat{TOP} = \alpha$ as shown in the diagram alongside.



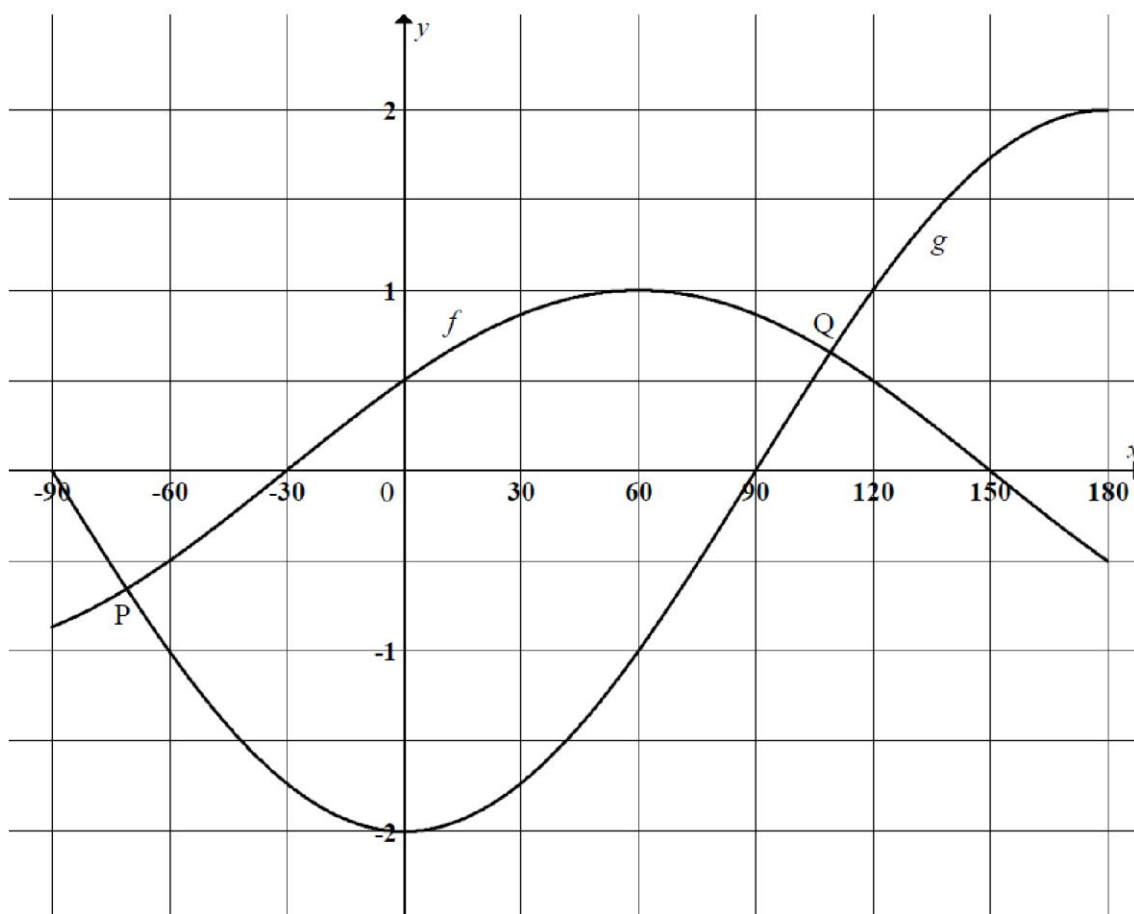
- 8.1.1 Determine the value of k . (2)
- 8.1.2 Write down the value of $\cos \alpha$. (1)
- 8.1.3 If it is further given that $\alpha + \beta = 180^\circ$, determine $\cos \beta$. (2)
- 8.1.4 Hence, determine the value of $\sin(\beta - \alpha)$. (4)
- 8.2 Consider the expression: $\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$
- 8.2.1 Prove that: $\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$ (4)
- 8.2.2 The above expression is undefined if $\sin 2x - \cos x = 0$. Solve this equation in the interval $0^\circ \leq x \leq 360^\circ$. (4)
- [17]

QUESTION 9

- 9.1 Simplify as far as possible: $\frac{\sin^2 \theta}{\sin(180^\circ - \theta) \cdot \cos(90^\circ + \theta) + \tan 45^\circ}$ (5)
- 9.2 Simplify without the use of a calculator: $\frac{\sin 104^\circ (2 \cos^2 15^\circ - 1)}{\tan 38^\circ \cdot \sin^2 412^\circ}$ (8)
- [13]

QUESTION 10

The graphs of $f(x) = \sin(x + 30^\circ)$ and $g(x) = -2\cos x$ for $-90^\circ \leq x \leq 180^\circ$ are given below. The graphs intersect at point P and point Q.

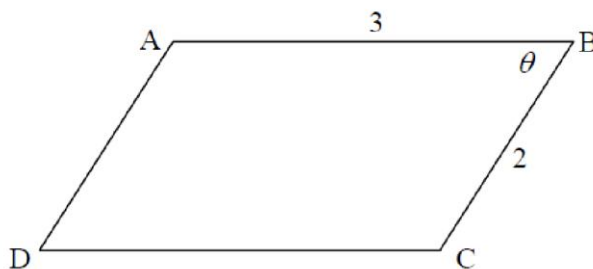


- 10.1 Calculate $f(0) - g(0)$. (1)
- 10.2 Calculate the x -coordinates of point P and point Q. (7)
- 10.3 For which values of x will $f(x) \geq g(x)$? (2)
- 10.4 Graph h is obtained by the following transformation of f : $h(x) = 2f(x + 60^\circ)$. Describe the relationship between g and h . (2)

[12]

QUESTION 11

ABCD is a parallelogram with $AB = 3$ units, $BC = 2$ units and $\hat{ABC} = \theta$ for $0^\circ < \theta \leq 90^\circ$.

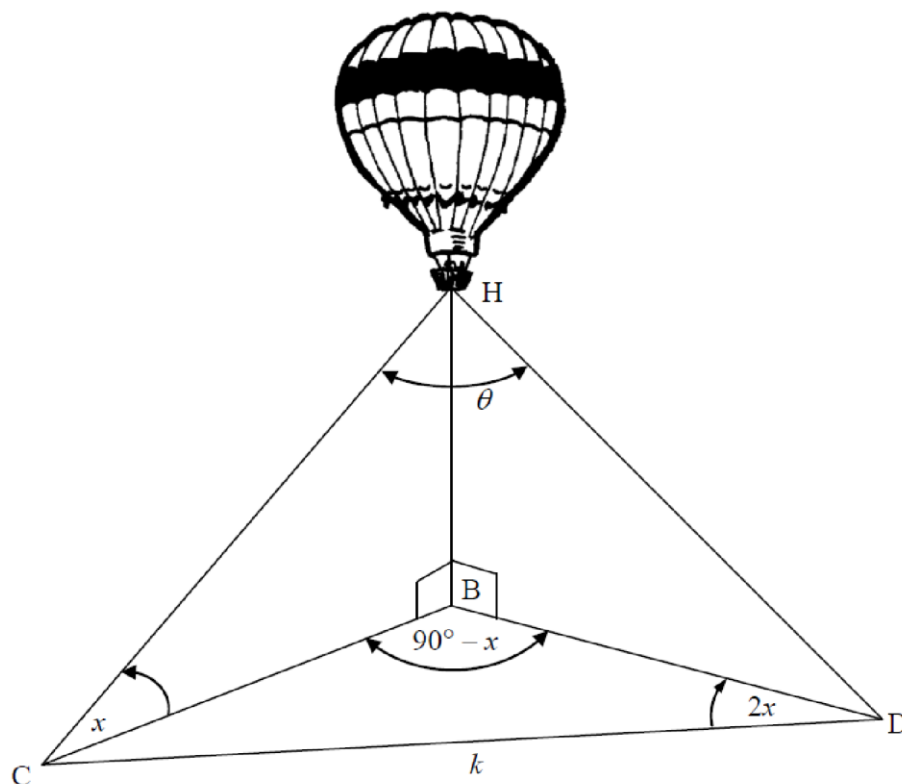


- 11.1 Prove that the area of parallelogram ABCD is $6 \sin \theta$. (3)
- 11.2 Calculate the value of θ for which the area of the parallelogram is $3\sqrt{3}$ square units. (3)
- 11.3 Determine the value of θ for which the parallelogram has the maximum area. (2)

[8]

QUESTION 12

A hot-air balloon H is directly above point B on the ground. Two ropes are used to keep the hot-air balloon in position. The ropes are held by two people on the ground at point C and point D . B , C and D are in the same horizontal plane. The angle of elevation from C to H is x . $\hat{CDB} = 2x$ and $\hat{CBD} = 90^\circ - x$. The distance between C and D is k metres.



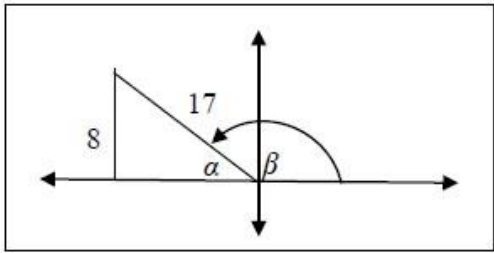
12.1 Show that $CB = 2k \sin x$. (5)

12.2 Hence, show that the length of rope HC is $2k \tan x$. (3)

12.3 If $k = 40$ m, $x = 23^\circ$ and $HD = 31,8$ m, calculate θ , the angle between the two ropes. (4)
[12]

QUESTION 8

No calculator allowed in this question

8.1.1	$OT = k$, $PT = 8$ and $OP = 17$ $k^2 + 8^2 = 17^2$ $k^2 = 289 - 64$ $k^2 = 225$ $k = \pm 15$ $k > 0$ $k = 15$ OR $k^2 = 17^2 - 8^2$ $k^2 = (17 - 8)(17 + 8)$ $= 25 \times 9$ $= 225$ $k = \pm 15$ $k > 0$ $k = 15$	✓ substitution into Pythagoras ✓ $k = 15$ (2) ✓ substitution into Pythagoras ✓ $k = 15$ (2)
8.1.2	$\cos \alpha = \frac{15}{17}$	✓ $\frac{15}{17}$ (1)
8.1.3	$\alpha + \beta = 180^\circ$ $\beta = 180^\circ - \alpha$ $\therefore \cos \beta = \cos(180^\circ - \alpha)$ $= -\cos \alpha$ $= -\frac{15}{17}$ OR  $\therefore \cos \beta = \cos(180^\circ - \alpha)$ $= -\cos \alpha$ $= -\frac{15}{17}$	✓ $\cos(180^\circ - \alpha)$ or $-\cos \alpha$ ✓ $-\frac{15}{17}$ (2) ✓ $\cos(180^\circ - \alpha)$ or $-\cos \alpha$ ✓ $-\frac{15}{17}$ (2)

8.1.4	$\begin{aligned} \sin(\beta - \alpha) &= \sin \beta \cos \alpha - \cos \beta \sin \alpha \\ &= \left(\frac{8}{17}\right)\left(\frac{15}{17}\right) - \left(-\frac{15}{17}\right)\left(\frac{8}{17}\right) \\ &= \frac{120}{289} + \frac{120}{289} \\ &= \frac{240}{289} \end{aligned}$ OR $\begin{aligned} \beta - \alpha &= (180^\circ - \alpha) - \alpha \\ &= 180^\circ - 2\alpha \\ \sin(\beta - \alpha) &= \sin(180^\circ - 2\alpha) \\ &= \sin 2\alpha \\ &= 2 \sin \alpha \cos \alpha \\ &= 2 \left(\frac{8}{17}\right)\left(\frac{15}{17}\right) \\ &= \frac{240}{289} \end{aligned}$	✓ expansion ✓ $\sin \beta = \frac{8}{17}$ ✓ $\sin \alpha = \frac{8}{17}$ ✓ $\frac{240}{289}$ (4) ✓ substitute β ✓ $2 \sin \alpha \cos \alpha$ ✓ $\sin \alpha = \frac{8}{17}$ ✓ $\frac{240}{289}$ (4)
8.2.1	$\begin{aligned} LHS &= \frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} \\ &= \frac{1 - (1 - 2 \sin^2 x) - \sin x}{2 \sin x \cos x - \cos x} \\ &= \frac{2 \sin^2 x - \sin x}{2 \sin x \cos x - \cos x} \\ &= \frac{\sin x(2 \sin x - 1)}{\cos x(2 \sin x - 1)} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \\ &= RHS \end{aligned}$ OR	✓ $1 - 2 \sin^2 x$ ✓ $2 \sin x \cos x$ ✓ either $\sin x(2 \sin x - 1)$ or $\cos x(2 \sin x - 1)$ ✓ $\frac{\sin x}{\cos x}$ (4)

$$\begin{aligned}
 LHS &= \frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} \\
 &= \frac{1 - (2 \cos^2 x - 1) - \sin x}{2 \sin x \cos x - \cos x} \\
 &= \frac{2 - \cos^2 x - \sin x}{2 \sin x \cos x - \cos x} \\
 &= \frac{2(1 - \cos^2 x) - \sin x}{2 \sin x \cos x - \cos x} \\
 &= \frac{2 \sin^2 x - \sin x}{2 \sin x \cos x - \cos x} \\
 &= \frac{\sin x(2 \sin x - 1)}{\cos x(2 \sin x - 1)} \\
 &= \frac{\sin x}{\cos x} \\
 &= \tan x \\
 &= RHS
 \end{aligned}$$

OR

$$\begin{aligned}
 LHS &= \frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} \\
 &= \frac{1 - (\cos^2 x - \sin^2 x) - \sin x}{2 \sin x \cos x - \cos x} \\
 &= \frac{1 - \cos^2 x + \sin^2 x - \sin x}{2 \sin x \cos x - \cos x} \\
 &= \frac{\sin^2 x + \sin^2 x - \sin x}{2 \sin x \cos x - \cos x} \\
 &= \frac{2 \sin^2 x - \sin x}{2 \sin x \cos x - \cos x} \\
 &= \frac{\sin x(2 \sin x - 1)}{\cos x(2 \sin x - 1)} \\
 &= \frac{\sin x}{\cos x} \\
 &= \tan x \\
 &= RHS
 \end{aligned}$$

$$\begin{aligned}
 &\checkmark 2 \cos^2 x - 1 \\
 &\checkmark 2 \sin x \cos x
 \end{aligned}$$

$$\begin{aligned}
 &\checkmark \\
 &\text{either } \sin x(2 \sin x - 1) \\
 &\text{or} \\
 &\cos x(2 \sin x - 1) \\
 &\checkmark \frac{\sin x}{\cos x}
 \end{aligned}$$

(4)

$$\begin{aligned}
 &\checkmark \cos^2 x - \sin^2 x \\
 &\checkmark 2 \sin x \cos x
 \end{aligned}$$

$$\begin{aligned}
 &\checkmark \\
 &\text{either } \sin x(2 \sin x - 1) \\
 &\text{or} \\
 &\cos x(2 \sin x - 1) \\
 &\checkmark \frac{\sin x}{\cos x}
 \end{aligned}$$

(4)

8.2.2	$\sin 2x - \cos x = 0$ $2 \sin x \cos x - \cos x = 0$ $\cos x(2 \sin x - 1) = 0$ $\cos x = 0$ $x = 90^\circ + 360^\circ k \quad \text{or} \quad x = 270^\circ + 360^\circ k \quad k \in \mathbb{Z}$ or $\sin x = \frac{1}{2}$ $x = 30^\circ + 360^\circ k \quad \text{or} \quad x = 150^\circ + 360^\circ k$ $x = 90^\circ \text{ or } x = 270^\circ \text{ or } x = 30^\circ \text{ or } x = 150^\circ$ OR $\sin 2x = \cos x$ $\sin 2x = \sin(90^\circ - x)$ $2x = 90^\circ - x + 360^\circ k; k \in \mathbb{Z} \quad \text{or} \quad 2x = 180^\circ - (90^\circ - x) + 360^\circ k$ $3x = 90^\circ + 360^\circ k \quad 2x = 90^\circ + x + 360^\circ k$ $x = 30^\circ + 120^\circ k \quad x = 90^\circ + 360^\circ k$ $x = 30^\circ \text{ or } x = 150^\circ \text{ or } x = 270^\circ \text{ or } x = 90^\circ$	$\checkmark 2 \sin x \cos x$ $\checkmark \begin{cases} \cos x = 0 \\ \text{and} \\ \sin x = \frac{1}{2} \end{cases}$ $\checkmark \text{ for two correct answers}$ $\checkmark \text{ for four correct answers}$ (4) $\checkmark \sin(90^\circ - x)$ $\checkmark x = 30^\circ + 120^\circ k$ and $x = 90^\circ + 360^\circ k$ $\checkmark \text{ for two correct answers}$ $\checkmark \text{ for four correct answers}$ (4) [17]
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QUESTION 9

9.1	$\frac{\sin^2 \theta}{\sin(180^\circ - \theta) \cdot \cos(90^\circ + \theta) + \tan 45^\circ}$ $= \frac{\sin^2 \theta}{(\sin \theta)(-\sin \theta) + 1}$ $= \frac{\sin^2 \theta}{-\sin^2 \theta + 1}$ $= \frac{\sin^2 \theta}{\cos^2 \theta}$ $= \tan^2 \theta$	$\checkmark \sin \theta$ $\checkmark -\sin \theta$ $\checkmark 1$ $\checkmark \cos^2 \theta$ $\checkmark \tan^2 \theta$
9.2	$\frac{\sin 104^\circ (2 \cos^2 15^\circ - 1)}{\tan 38^\circ \sin^2 412^\circ}$ $= \frac{\sin 76^\circ \cdot \cos 30^\circ}{\tan 38^\circ \cdot (\sin 52^\circ)^2}$ $= \frac{2 \sin 38^\circ \cos 38^\circ \left(\frac{\sqrt{3}}{2} \right)}{\left(\frac{\sin 38^\circ}{\cos 38^\circ} \right) (\cos 38^\circ)^2}$ $= \frac{\sqrt{3} \sin 38^\circ \cos 38^\circ}{\sin 38^\circ \cos 38^\circ}$ $= \sqrt{3}$ NOTE: - If $\cos 30^\circ$ is missing: deduct 1 mark - Answer only: 0/8 OR $\frac{\sin 104^\circ (2 \cos^2 15^\circ - 1)}{\tan 38^\circ \sin^2 412^\circ}$ $= \frac{\sin 2(52^\circ) \cdot (2 \cos^2 15^\circ - 1)}{\frac{\sin 38^\circ}{\cos 38^\circ} \cdot (\sin 52^\circ)^2}$ $= \frac{2 \sin 52^\circ \cos 52^\circ \cdot \cos 30^\circ}{\left(\frac{\cos 52^\circ}{\sin 52^\circ} \right) (\sin 52^\circ)^2}$ $= 2 \cos 30^\circ$ $= 2 \cdot \frac{\sqrt{3}}{2}$ $= \sqrt{3}$	$\checkmark \sin 76^\circ$ $\checkmark \cos 30^\circ$ $\checkmark \frac{\sin 38^\circ}{\cos 38^\circ}$ $\checkmark \sin 52^\circ$ $\checkmark 2 \sin 38^\circ \cos 38^\circ$ $\checkmark \frac{\sqrt{3}}{2}$ $\checkmark$ $\sin 52^\circ = \cos 38^\circ$ $\checkmark \sqrt{3}$ $\checkmark \sin 2(52^\circ)$ $\checkmark \frac{\sin 38^\circ}{\cos 38^\circ}$ $\checkmark \sin 52^\circ$ $\checkmark 2 \sin 52^\circ \cos 52^\circ$ $\checkmark \cos 30^\circ$ $\checkmark$ $\cos 52^\circ = \sin 38^\circ$ and $\sin 52^\circ = \cos 38^\circ$ $\checkmark \frac{\sqrt{3}}{2}$ $\checkmark \sqrt{3}$

OR

$$\begin{aligned} & \frac{\sin 104^\circ (2 \cos^2 15^\circ - 1)}{\tan 38^\circ \sin^2 412^\circ} \\ &= \frac{\sin 104^\circ \cdot \cos 30^\circ}{\left(\frac{\sin 38^\circ}{\cos 38^\circ}\right) (\sin 52^\circ)^2} \\ &= \frac{(\sin 104^\circ) \left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sin 38^\circ}{\cos 38^\circ}\right) (\cos^2 38^\circ)} \\ &= \frac{\sqrt{3} \sin 104^\circ}{2 \sin 38^\circ \cos 38^\circ} \\ &= \frac{\sqrt{3} \sin 104^\circ}{\sin 76^\circ} \\ &= \frac{\sqrt{3} \sin 76^\circ}{\sin 76^\circ} \quad \text{or} \quad \frac{\sqrt{3} \cos 14^\circ}{\cos 14^\circ} \\ &= \sqrt{3} \end{aligned}$$

OR

$$\begin{aligned} & \frac{\sin 104^\circ (2 \cos^2 15^\circ - 1)}{\tan 38^\circ \sin^2 412^\circ} \\ &= \frac{\sin 104^\circ \cdot \cos 30^\circ}{\frac{\sin 38^\circ}{\cos 38^\circ} \cdot (\sin 52^\circ)^2} \\ &= \frac{\sin 104^\circ \cdot \frac{\sqrt{3}}{2}}{\left(\frac{\cos 52^\circ}{\sin 52^\circ}\right) (\sin 52^\circ)^2} \\ &= \frac{\sin 104^\circ \cdot \frac{\sqrt{3}}{2}}{\cos 52^\circ (\sin 52^\circ)} \\ &= \frac{\sin 104^\circ \cdot \frac{\sqrt{3}}{2}}{\frac{1}{2} \sin 104^\circ} \\ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} & \checkmark \cos 30^\circ \\ & \checkmark \frac{\sin 38^\circ}{\cos 38^\circ} \\ & \checkmark \sin 52^\circ \end{aligned}$$

$$\begin{aligned} & \checkmark \cos^2 38^\circ \\ & \checkmark \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} & \checkmark \checkmark \sin 76^\circ \\ & \checkmark \sqrt{3} \end{aligned}$$

(8)

$$\begin{aligned} & \checkmark \cos 30^\circ \\ & \checkmark \frac{\sin 38^\circ}{\cos 38^\circ} \\ & \checkmark \sin 52^\circ \\ & \checkmark \frac{\sqrt{3}}{2} \\ & \checkmark \\ & \cos 52^\circ = \sin 38^\circ \\ & \text{and} \\ & \sin 52^\circ = \cos 38^\circ \\ & \checkmark \cos 52^\circ \cdot \sin 52^\circ \\ & \checkmark \frac{1}{2} \sin 104^\circ \\ & \checkmark \sqrt{3} \end{aligned}$$

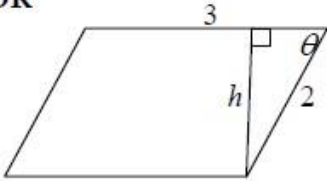
(8)

[13]

QUESTION 10

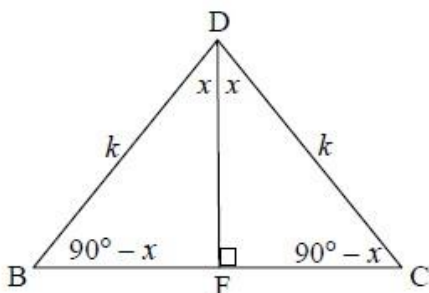
10.1	$f(0) - g(0) = 0,5 - (-2) = 2,5$	✓ 2,5 (1)
10.2	$\sin(x + 30^\circ) = -2 \cos x$ $\sin x \cdot \cos 30^\circ + \cos x \cdot \sin 30^\circ = -2 \cos x$ $\left(\frac{\sqrt{3}}{2}\right) \sin x + \left(\frac{1}{2}\right) \cos x = -2 \cos x$ $\sqrt{3} \sin x + \cos x = -4 \cos x$ $\sqrt{3} \sin x = -5 \cos x$ $\tan x = -\frac{5}{\sqrt{3}}$ $x = 109,11^\circ + 180^\circ k ; k \in \mathbb{Z}$ $x_P = -70,89^\circ$ and $x_Q = 109,11^\circ$ OR $\sin(x + 30^\circ) = -2 \cos x$ $\cos(90^\circ - x - 30^\circ) = -2 \cos x$ $\cos(60^\circ - x) = -2 \cos x$ $\cos 60^\circ \cos x + \sin 60^\circ \sin x = -2 \cos x$ $\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x = -2 \cos x$ $\cos x + \sqrt{3} \sin x = -4 \cos x$ $\sqrt{3} \sin x = -5 \cos x$ $\tan x = -\frac{5}{\sqrt{3}}$ $x = 109,11^\circ + 180^\circ k ; k \in \mathbb{Z}$ $x_P = -70,89^\circ$ and $x_Q = 109,11^\circ$	✓ equation ✓ expansion of $\sin(x + 30^\circ)$ ✓ substitution of special angles ✓ simplification ✓ $\tan x = -\frac{5}{\sqrt{3}}$ ✓ $x_P = -70,89^\circ$ ✓ $x_Q = 109,11^\circ$ (7) ✓ equation ✓ expansion of $\cos(60^\circ - x)$ ✓ substitution of special angles ✓ simplification ✓ $\tan x = -\frac{5}{\sqrt{3}}$ ✓ $x_P = -70,89^\circ$ ✓ $x_Q = 109,11^\circ$ (7)
10.3	$-70,89^\circ \leq x \leq 109,11^\circ$ OR $[-70,89^\circ ; 109,11^\circ]$ OR $x_P \leq x \leq x_Q$	✓ angles ✓ correct interval (2)
10.4	$h(x) = 2 \sin(x + 60^\circ + 30^\circ) = 2 \sin(x + 90^\circ) = 2 \cos x = -g(x)$ h is the reflection of g about the x-axis. OR f is shifted to the left through 60° and then doubled. $\therefore h$ is the reflection of g about the x-axis.	✓✓ reflection about the x-axis or line $y = 0$ (2) ✓✓ reflection about the x-axis or line $y = 0$ (2) [12]

QUESTION 11

11.1	Area parallelogram ABCD = $2 \times \text{Area } \triangle ABC$ $= 2 \left[\left(\frac{1}{2} \right) (3)(2) \sin \theta \right]$ $= 6 \sin \theta$ OR  NOTE: If no working is shown, then 0/3 $\frac{h}{2} = \sin \theta$ $h = 2 \sin \theta$ $\therefore \text{Area } ABCD = \text{base} \times \text{height} = 3h = 3 \cdot 2 \sin \theta = 6 \sin \theta$ OR Area of parallelogram ABCD = area of $\triangle ABC$ + area of $\triangle ADC$ $= \left(\frac{1}{2} \right) (3)(2) \sin \theta + \left(\frac{1}{2} \right) (3)(2) \sin \theta$ $= 6 \sin \theta$ OR $\text{Area} = \frac{1}{2} (\text{sum of // sides}) \times h$ $= \frac{1}{2} (3 + 3) \times 2 \sin \theta$ $= 6 \sin \theta$	✓✓ 2area $\triangle ABC$ ✓ substitution into area rule (3) ✓ $\frac{h}{2} = \sin \theta$ ✓ $h = 2 \sin \theta$ ✓ $b \cdot h$ (3) ✓ sum of areas ✓ equal sides and equal angles (3) ✓ formula ✓ $h = 2 \sin \theta$ ✓ substitution (3)
11.2	Area of parallelogram ABCD = $3\sqrt{3}$ $6 \sin \theta = 3\sqrt{3}$ $\sin \theta = \frac{\sqrt{3}}{2}$ $\theta = 60^\circ$ NOTE: Deduct 1 mark if both 60° and 120° are given as answers OR $6 \sin 60^\circ = 3\sqrt{3}$ $\therefore \theta = 60^\circ$	✓ $6 \sin \theta = 3\sqrt{3}$ ✓ $\sin \theta = \frac{\sqrt{3}}{2}$ ✓ 60° (3) ✓✓ $6 \sin \theta = 3\sqrt{3}$ ✓ 60° (3)
11.3	Maximum area of parallelogram occurs when $\sin \theta = 1$, that is when $\theta = 90^\circ$	✓ $\sin \theta = 1$ ✓ $\theta = 90^\circ$ (2)

[8]

QUESTION 12

12.1	$\frac{CB}{\sin \hat{BDC}} = \frac{CD}{\sin \hat{CBD}}$ $\frac{CB}{\sin 2x} = \frac{k}{\sin(90^\circ - x)}$ $CB = \frac{k \cdot \sin 2x}{\sin(90^\circ - x)}$ $CB = \frac{k \cdot 2 \sin x \cos x}{\cos x}$ $= 2k \sin x$ OR $\hat{DCB} = 180^\circ - (90^\circ - x + 2x) = 90^\circ - x$ $\therefore DC = DB = k$  Draw $DF \perp BC$ $\frac{CF}{CD} = \sin x$ $CF = k \sin x$ $CB = 2CF$ $CB = 2k \sin x$ OR $\hat{DCB} = 180^\circ - (90^\circ - x + 2x) = 90^\circ - x$ $\therefore DC = DB = k$ $CB^2 = CD^2 + BD^2 - 2 \cdot CD \cdot BD \cdot \cos 2x$ $CB^2 = k^2 + k^2 - 2k^2 \cos 2x$ $= 2k^2(1 - \cos 2x)$ $= 2k^2(1 - (1 - 2 \sin^2 x))$ $= 2k^2(2 \sin^2 x)$ $= 4k^2 \sin^2 x$ $= (2k \sin x)^2$ $CB = 2k \sin x$	✓ Using the sine rule in triangle CBD ✓ $\frac{CB}{\sin 2x} = \frac{k}{\sin(90^\circ - x)}$ ✓ $\frac{k \cdot \sin 2x}{\sin(90^\circ - x)}$ ✓ $2 \sin x \cdot \cos x$ ✓ $\cos x$ (5) ✓ $\hat{DCB} = \hat{DBC} = 90^\circ - x$ ✓ $DC = DB = k$ ✓ $\hat{CDF} = x$ ✓ $CF = k \sin x$ ✓ $CB = 2CF$ (5) ✓ $\hat{DCB} = \hat{DBC} = 90^\circ - x$ ✓ $DC = DB = k$ ✓ using cosine rule in triangle CDB ✓ factors ✓ simplification (5)
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12.2	$\cos x = \frac{BC}{HC}$ $HC = \frac{BC}{\cos x}$ $= \frac{2k \sin x}{\cos x}$ $= 2k \tan x$ OR $\frac{HC}{\sin 90^\circ} = \frac{BC}{\sin(90^\circ - x)}$ $HC = \frac{BC}{\sin(90^\circ - x)}$ $= \frac{2k \sin x}{\cos x}$ $= 2k \tan x$	$\checkmark \cos x = \frac{BC}{HC}$ $\checkmark HC = \frac{BC}{\cos x}$ ✓ substitution of BC (3) $\checkmark HC = \frac{BC}{\sin(90^\circ - x)}$ ✓ substitution of BC ✓ $\sin(90^\circ - x) = \cos x$ (3)
12.3	$HC = 2k \tan x = 2(40) \cdot \tan(23^\circ) = 33,9579...$ In $\triangle HCD$: $CD^2 = HC^2 + HD^2 - 2HC \cdot HD \cdot \cos \theta$ $\cos \theta = \frac{HC^2 + HD^2 - CD^2}{2HC \cdot HD}$ $= \frac{(33,9579...) ^2 + 31,8^2 - 40^2}{2(33,9579...)(31,8)}$ $\cos \theta = 0,2613...$ $\therefore \theta = 74,85^\circ$	✓ value of HC ✓ substitution into cos formula ✓ $\cos \theta = 0,2613...$ ✓ $74,85^\circ$ (4) [12]

QUESTION 8

8.1 Simplify as far as possible: $1 - \sin^2 \theta + 3 - \cos^2 \theta$ (2)

8.2 Simplify WITHOUT the use of a calculator: $\sqrt{4^{\sin 150^\circ} \times 2^{3 \tan 225^\circ}}$ (4)

8.3 Prove that $\frac{\cos^2 x \sin^2 x + \cos^4 x}{1 - \sin x} = 1 + \sin x$ (4)

8.4 Prove that for any angle θ , $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$.
(Hint: $3\theta = \theta + 2\theta$) (4)

8.5 If $x = \cos 20^\circ$, use QUESTION 8.4 to show that $8x^3 - 6x - 1 = 0$. (2)
[16]

QUESTION 9

9.1 Simplify to ONE trigonometric function WITHOUT using a calculator:

$$\frac{\cos 160^\circ \tan 200^\circ}{2 \sin(-10^\circ)} \quad (6)$$

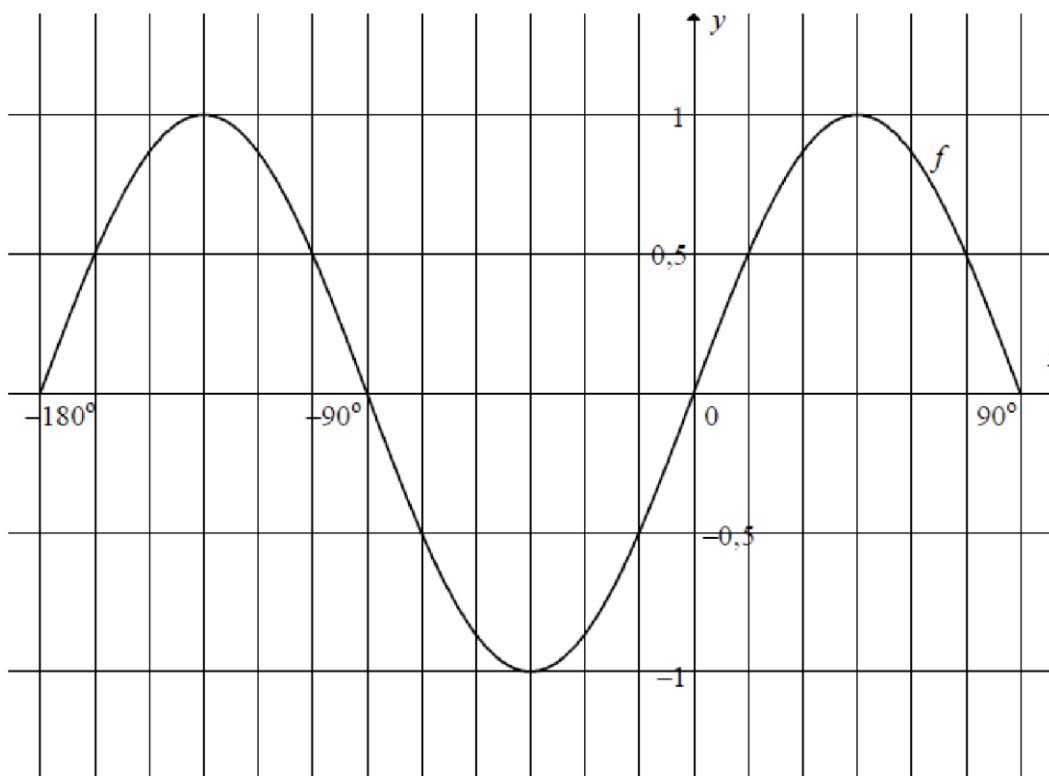
9.2 Consider $\cos(x + 45^\circ) \cos(x - 45^\circ)$.

9.2.1 Show that $\cos(x + 45^\circ) \cos(x - 45^\circ) = \frac{1}{2} \cos 2x$. (4)

9.2.2 Hence, determine a value of x in the interval $0^\circ \leq x \leq 180^\circ$ for which $\cos(x + 45^\circ) \cos(x - 45^\circ)$ is a minimum. (3)
[13]

QUESTION 10

The graph of $f(x) = \sin 2x$ for $-180^\circ \leq x \leq 90^\circ$ is shown in the sketch below.

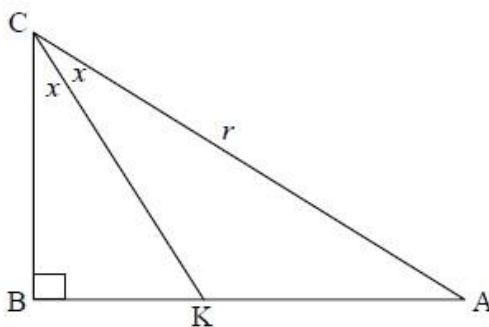


- 10.1 Write down the range of f . (2)
- 10.2 Determine the period of $f\left(\frac{3}{2}x\right)$. (2)
- 10.3 Draw the graph of $g(x) = \cos(x - 30^\circ)$ for $-180^\circ \leq x \leq 90^\circ$ on the system of axes on DIAGRAM SHEET 3. Clearly label ALL x -intercepts and turning points. (4)
- 10.4 Hence, or otherwise, determine the values of x in the interval $-180^\circ \leq x \leq 90^\circ$ for which $f(x) \cdot g(x) < 0$. (4)
- 10.5 Describe the transformation that graph f has to undergo to form $y = \sin(2x + 60^\circ)$. (2)
- 10.6 Determine the general solution of $\sin 2x = \cos(x - 30^\circ)$. (6)

[20]

QUESTION 11

In the diagram below, ABC is a right-angled triangle. KC is the bisector of $\hat{ACB}$.
 $AC = r$ units and $\hat{BCK} = x$.



11.1 Write down AB in terms of x and r . (2)

11.2 Give the size of $\hat{AKC}$ in terms of x . (1)

11.3 If it is given that $\frac{AK}{AB} = \frac{2}{3}$, calculate the value of x . (8)
 [11]

QUESTION 8

8.1	$1 - \sin^2 \theta + 3 - \cos^2 \theta$ $= 4 - (\sin^2 \theta + \cos^2 \theta)$ $= 3$ OR $\cos^2 \theta + 3 - \cos^2 \theta$ $= 3$	✓ simplification ✓ 3 (2)
8.2	$\sqrt{4^{\sin 150^\circ} \cdot 2^{3 \tan 225^\circ}}$ $= \sqrt{4^{\sin 30^\circ} \cdot 2^{3 \tan 45^\circ}}$ $= \sqrt{(2^2)^{\frac{1}{2}} \cdot 2^3}$ $= \sqrt{16}$ $= 4$ OR $\sin 150^\circ = \frac{1}{2}$ $\tan 225^\circ = 1$ $\sqrt{4^{\sin 150^\circ} \cdot 2^{3 \tan 225^\circ}}$ $= \sqrt{4^{\frac{1}{2}} \cdot 2^3}$ $= \sqrt{2 \cdot 2^3}$ $= \sqrt{16}$ $= 4$	✓ rewrite using reduction formula ✓ substituting special angles ✓ simplification ✓ 4 (4)
8.3	$LHS = \frac{\cos^2 x (\sin^2 x + \cos^2 x)}{1 - \sin x}$ $= \frac{\cos^2 x \cdot (1)}{1 - \sin x}$ $= \frac{(1 - \sin^2 x)}{1 - \sin x}$ $= \frac{(1 + \sin x)(1 - \sin x)}{1 - \sin x}$ $= 1 + \sin x$ $= RHS$	✓ factorisation ✓ 1 ✓ $1 - \sin^2 x$ ✓ factors (4)

8.4	$\cos 3\theta$ $= \cos(2\theta + \theta)$ $= \cos 2\theta \cdot \cos \theta - \sin 2\theta \cdot \sin \theta$ $= (2\cos^2 \theta - 1) \cdot \cos \theta - 2\sin \theta \cdot \cos \theta \cdot \sin \theta$ $= 2\cos^3 \theta - \cos \theta - 2\sin^2 \theta \cdot \cos \theta$ $= 2\cos^3 \theta - \cos \theta - 2(1 - \cos^2 \theta) \cdot \cos \theta$ $= 2\cos^3 \theta - \cos \theta - 2\cos \theta + 2\cos^3 \theta$ $= 4\cos^3 \theta - 3\cos \theta$	$\checkmark$ expansion $\checkmark 2\cos^2 \theta - 1$ $\checkmark 2\sin \theta \cdot \cos \theta$ $\checkmark 1 - \cos^2 \theta$
8.5	$\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$ $\cos 3(20^\circ) = 4\cos^3(20^\circ) - 3\cos(20^\circ)$ $\frac{1}{2} = 4x^3 - 3x$ $8x^3 - 6x - 1 = 0$	$\checkmark \theta = 20^\circ$ $\checkmark \cos 60^\circ = \frac{1}{2}$
		(4)
		(2)
		[16]

QUESTION 9

9.1	$\frac{\cos 160^\circ \cdot \tan 200^\circ}{2 \sin(-10^\circ)}$ $= \frac{(-\cos 20^\circ)(\tan 20^\circ)}{2(-\sin 10^\circ)}$ $= \frac{(-\cos 20^\circ)\left(\frac{\sin 20^\circ}{\cos 20^\circ}\right)}{-2 \sin 10^\circ}$ $= \frac{2 \sin 10^\circ \cos 10^\circ}{2 \sin 10^\circ}$ $= \cos 10^\circ$	✓ $-\cos 20^\circ$ ✓ $\tan 20^\circ$ ✓ $-\sin 10^\circ$ ✓ $\frac{\sin 20^\circ}{\cos 20^\circ}$ ✓ $2 \sin 10^\circ \cos 10^\circ$ ✓ $\cos 10^\circ$ (6)
9.2.1	$LHS = \cos(x + 45^\circ) \cdot \cos(x - 45^\circ)$ $= (\cos x \cos 45^\circ - \sin x \sin 45^\circ)(\cos x \cos 45^\circ + \sin x \sin 45^\circ)$ $= \cos^2 x \cos^2 45^\circ - \sin^2 x \sin^2 45^\circ$ $= \cos^2 x \left(\frac{\sqrt{2}}{2}\right)^2 - \sin^2 x \left(\frac{\sqrt{2}}{2}\right)^2 \text{ or } \left[\cos^2 x \left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2 x \left(\frac{1}{\sqrt{2}}\right)^2\right]$ $= \frac{1}{2} \cos^2 x - \frac{1}{2} \sin^2 x$ $= \frac{1}{2} (\cos^2 x - \sin^2 x)$ $= \frac{1}{2} \cos 2x$ OR $2 \cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$ $\cos \alpha \cos \beta = \frac{1}{2} (\cos(\alpha + \beta) + \cos(\alpha - \beta))$ Let $\alpha = x + 45^\circ$ and $\beta = x - 45^\circ$ $\therefore \cos(x + 45^\circ) \cos(x - 45^\circ)$ $= \frac{1}{2} (\cos((x + 45^\circ) + (x - 45^\circ)) + \cos((x + 45^\circ) - (x - 45^\circ)))$ $= \frac{1}{2} (\cos 2x + \cos 90^\circ)$ $= \frac{1}{2} \cos 2x$	✓ expand $\cos(x + 45^\circ)$ ✓ expand $\cos(x - 45^\circ)$ ✓ substitute special angles ✓ simplification (4) ✓✓ deriving identity ✓ substitution ✓ simplification (4)

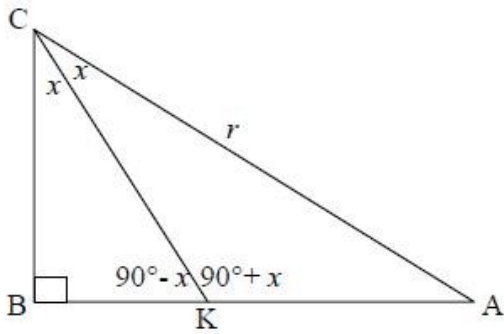
9.2.2	$\cos(x + 45^\circ)\cos(x - 45^\circ)$ has a minimum when $\frac{1}{2}\cos 2x$ has a minimum. The minimum value of $\cos 2x$ is -1 $\cos 2x = -1$ $2x = 180^\circ$ $x = 90^\circ$	✓ minimum value of -1 ✓ $2x = 180^\circ$ ✓ $x = 90^\circ$ (3) [13]
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QUESTION 10

10.1	Range = $[-1 ; 1]$	✓✓ $[-1 ; 1]$ (2)
10.2	$f\left(\frac{3}{2}x\right) = \sin 2\left(\frac{3}{2}x\right)$ $= \sin 3x$ $\therefore \text{Period} = \frac{360^\circ}{3} = 120^\circ$ OR $f\left(\frac{3}{2}x\right) = \sin 2\left(\frac{3}{2}x\right)$ $= \sin 3x$ $= \sin(3x + 360^\circ)$ $= \sin 3(x + 120^\circ)$ $\therefore \text{Period} = 120^\circ$	✓ $\sin 3x$ ✓ 120° (2) ✓ $\sin 3x$ ✓ 120° (2)

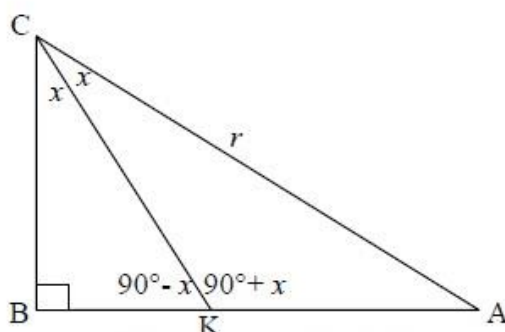
10.3		✓ x intercepts ✓✓ turning points ✓ shape (4)
10.4	$(-180^\circ; -90^\circ)$ or $(-60^\circ; 0^\circ)$ OR $-180^\circ < x < -90^\circ$ or $-60^\circ < x < 0^\circ$	✓ $> -180^\circ$ ✓ $< -90^\circ$ ✓ $> -60^\circ$ ✓ $< 0^\circ$ (4)
10.5	$y = \sin 2(x + 30^\circ)$ $\therefore$ translation of 30° to the left	✓ translation 30° ✓ to the left (2)
10.6	$\sin 2x = \cos(x - 30^\circ)$ $\sin 2x = \sin[90^\circ - (x - 30^\circ)]$ $= \sin(120^\circ - x)$ $2x = 120^\circ - x + 360^\circ k; k \in \mathbb{Z}$ $2x = 180^\circ - (120^\circ - x) + 360^\circ k$ $3x = 120^\circ + 360^\circ k$ or $2x - x = 60^\circ + 360^\circ k$ $x = 40^\circ + 120^\circ k; k \in \mathbb{Z}$ $x = 60^\circ + 360^\circ k; k \in \mathbb{Z}$ OR $\sin 2x = \cos(x - 30^\circ)$ $\cos(90^\circ - 2x) = \cos(x - 30^\circ)$ $90^\circ - 2x = x - 30^\circ + 360^\circ k$ or $90^\circ - 2x = 360^\circ - (x - 30^\circ) + 360^\circ k$ $-3x = -120^\circ + 360^\circ k$ $-x = 300^\circ + 360^\circ k$ $x = 40^\circ - 120^\circ k; k \in \mathbb{Z}$ $x = -300^\circ - 360^\circ k; k \in \mathbb{Z}$ $\therefore x = 40^\circ + 120^\circ k$ or $x = 60^\circ + 360^\circ k; k \in \mathbb{Z}$	✓ using co-function ✓ $2x = 120^\circ - x + 360^\circ k$ ✓ $x = 40^\circ + 120^\circ k$ ✓ $2x = 180^\circ - (120^\circ - x) + 360^\circ k$ $+ 360^\circ k$ ✓ $x = 60^\circ + 360^\circ k$ ✓ $k \in \mathbb{Z}$ (6) ✓ $\cos(90^\circ - x) = \cos(x - 30^\circ)$ ✓ $90^\circ - 2x = x - 30^\circ + 360^\circ k$ ✓ $x = 40^\circ - 120^\circ k$ ✓ $90^\circ - 2x = 360^\circ - (x - 30^\circ) + 360^\circ k$ ✓ $x = -300^\circ - 360^\circ k$ ✓ $k \in \mathbb{Z}$ (6)

QUESTION 11

11.1	$\frac{AB}{r} = \sin 2x$ $AB = r \sin 2x$	$\checkmark \frac{AB}{r} = \sin 2x$ $\checkmark AB = r \sin 2x$
11.2	$\hat{AKC} = 90^\circ + x$	$\checkmark \hat{AKC} = 90^\circ + x$
11.3	 In $\triangle AKC$: $\frac{\sin \hat{AKC}}{AC} = \frac{\sin \hat{ACK}}{AK}$ $\frac{\sin(90^\circ + x)}{r} = \frac{\sin x}{AK}$ $AK = \frac{r \sin x}{\sin(90^\circ + x)} = \frac{r \sin x}{\cos x}$ $\frac{AK}{AB} = \frac{2}{3}$ $\frac{\left(\frac{r \sin x}{\cos x}\right)}{r \sin 2x} = \frac{2}{3}$ $\frac{\sin x}{\cos x} = \frac{2}{3}$ $\frac{\sin x}{\cos x} \times \frac{1}{2 \sin x \cos x} = \frac{2}{3}$ $\frac{1}{2 \cos^2 x} = \frac{2}{3}$ $4 \cos^2 x = 3$ $\cos x = \frac{\sqrt{3}}{2}$ $x = 30^\circ$	$\checkmark \text{ sine rule}$ $\checkmark \text{ substitution}$ $\checkmark \text{ making AK subject of the formula}$ $\checkmark \cos x$ $\checkmark 2 \sin x \cdot \cos x$ $\checkmark \frac{1}{2 \cos^2 x}$ $\checkmark \cos x = \frac{\sqrt{3}}{2}$ $\checkmark x = 30^\circ$

OR

(8)



Using the sine-formula in $\triangle CBK$ and $\triangle CKA$:

$$\frac{\sin x}{BK} = \frac{\sin(90^\circ - x)}{BC} \quad \text{and} \quad \frac{\sin x}{KA} = \frac{\sin(90^\circ + x)}{AC}$$

$$\therefore \frac{BK}{BC} = \frac{KA}{AC}$$

$$\therefore \frac{1}{BC} = \frac{2}{r}$$

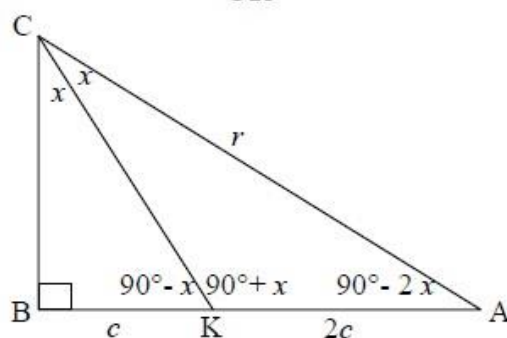
$$\therefore BC = \frac{1}{2}r$$

$$\therefore \cos 2x = \frac{BC}{AC} = \frac{\frac{1}{2}r}{r} = \frac{1}{2}$$

$$\therefore 2x = 60^\circ$$

$$\therefore x = 30^\circ$$

OR



$$\triangle CBK: KC = \frac{c}{\sin x}$$

$$\triangle CKA: \frac{\sin x}{2c} = \frac{\sin(90^\circ - 2x)}{KC} = \frac{\sin(90^\circ - 2x) \cdot \sin x}{c}$$

$$\therefore \sin(90^\circ - 2x) = \frac{1}{2} = \sin 30^\circ$$

$$\therefore 90^\circ - 2x = 30^\circ$$

$$x = 30^\circ$$

$$\checkmark \frac{\sin x}{BK} = \frac{\sin(90^\circ - x)}{BC}$$

$$\checkmark \frac{\sin x}{KA} = \frac{\sin(90^\circ + x)}{AC}$$

$$\checkmark \frac{BK}{BC} = \frac{KA}{AC}$$

$$\checkmark \frac{1}{BC} = \frac{2}{r}$$

$$\checkmark BC = \frac{1}{2}r$$

$$\checkmark \cos 2x = \frac{1}{2}$$

$$\checkmark 2x = 60^\circ$$

$$\checkmark x = 30^\circ$$

(8)

$$\checkmark KC = \frac{c}{\sin x}$$

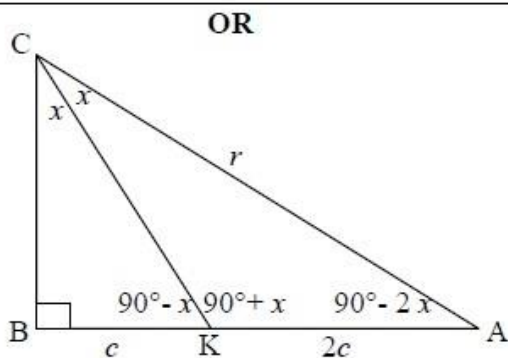
$$\checkmark \frac{\sin x}{2c} = \frac{\sin(90^\circ - 2x)}{KC}$$

✓✓ substitution

$$\checkmark \checkmark \sin(90^\circ - 2x) = \frac{1}{2}$$

$$\checkmark 90^\circ - 2x = 30^\circ$$

$$\checkmark x = 30^\circ$$



$\triangle CBK$:

$$\sin 2x = \frac{3c}{r} = 2 \sin x \cdot \cos x$$

$$\therefore r \sin x = \frac{3c}{2 \cos x} \dots\dots\dots(1)$$

$\triangle CKA$:

$$\frac{2c}{\sin x} = \frac{r}{\cos x}$$

$$\therefore r \sin x = 2c \cos x \dots\dots\dots(2)$$

Equate (1) and (2):

$$2c \cdot \cos x = \frac{3c}{2 \cos x}$$

$$\therefore \cos^2 x = \frac{3}{4}$$

$$\therefore \cos x = \frac{\sqrt{3}}{2}$$

$$\therefore x = 30^\circ$$

OR

(8)

$$\checkmark \sin 2x = \frac{3c}{r}$$

$$\checkmark 2 \sin x \cdot \cos x$$

$$\checkmark r \sin x = \frac{3c}{2 \cos x}$$

$$\checkmark \frac{2c}{\sin x} = \frac{r}{\cos x}$$

$$\checkmark r \sin x = 2c \cos x$$

$\checkmark$ equating

$$\checkmark \cos x = \frac{\sqrt{3}}{2}$$

$$\checkmark 30^\circ$$

(8)

$$\frac{AK}{KB} = \frac{2}{1} = 2$$

$$2 = \frac{\frac{1}{2}AK \cdot BC}{\frac{1}{2}BK \cdot BC}$$

$$= \frac{\text{area AKC}}{\text{area ABC}}$$

$$= \frac{\frac{1}{2}rCK \sin x}{\frac{1}{2}BC \cdot CK \sin x}$$

$$= \frac{r}{BC}$$

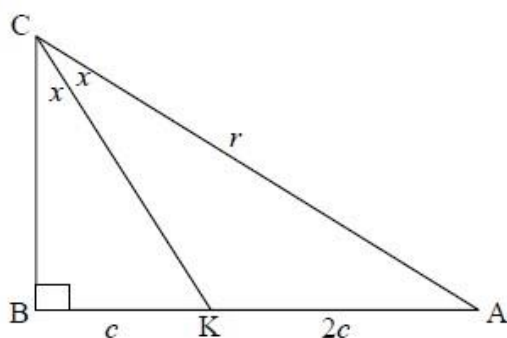
$$\therefore \frac{BC}{r} = \frac{1}{2}$$

$$\therefore \cos 2x = \frac{1}{2}$$

$$\therefore 2x = 60^\circ$$

$$\therefore x = 30^\circ$$

OR



By the Internal Bisector Theorem:

$$\frac{CB}{CA} = \frac{BK}{KA} = \frac{1}{2}$$

$$\cos 2x = \frac{1}{2}$$

$$2x = 60^\circ$$

$$x = 30^\circ$$

✓ multiplying by $\frac{1}{2}BC$

✓
area of triangles

✓
area formula in
triangles

$$\checkmark \frac{r}{BC} = 2$$

$$\checkmark \frac{BC}{r} = \frac{1}{2}$$

$$\checkmark \cos 2x = \frac{1}{2}$$

$$\checkmark 2x = 60^\circ$$

$$\checkmark x = 30^\circ$$

(8)

✓✓

For stating Internal
Bisector Theorem

$$\checkmark \checkmark \checkmark \frac{CB}{CA} = \frac{BK}{KA} = \frac{1}{2}$$

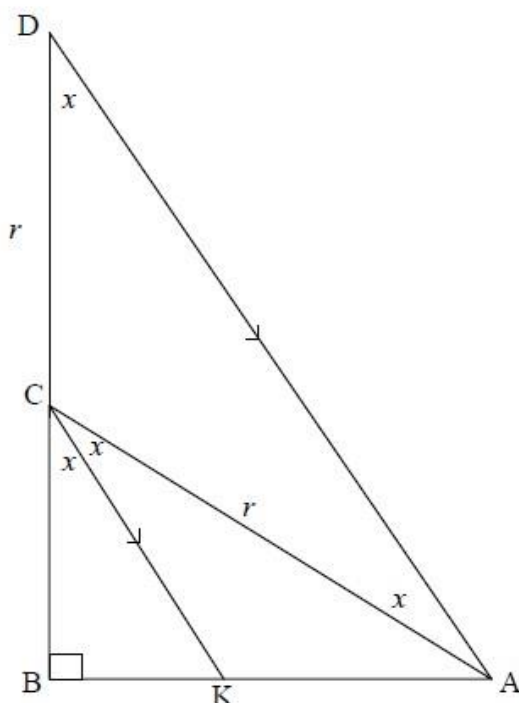
$$\checkmark \cos 2x = \frac{1}{2}$$

$$\checkmark 2x = 60^\circ$$

$$\checkmark x = 30^\circ$$

(8)

OR



Produce BC to D and draw CK parallel to DA.

$\hat{CAD} = \hat{KCA}$ and $\hat{BCK} = \hat{D}$

$$\therefore DC = CA = r$$

$$\therefore \triangle BKC \parallel \triangle BAD$$

$$\therefore \frac{BK}{BA} = \frac{BC}{BD} = 3$$

$$\therefore BD = 3BC = BC + r$$

$$\therefore BC = \frac{1}{2}r$$

$$\therefore \cos 2x = \frac{\frac{1}{2}r}{r} = \frac{1}{2}$$

$$\therefore 2x = 60^\circ$$

$$\therefore x = 30^\circ$$

$$\checkmark DC = CA = r$$

$$\checkmark \triangle BKC \parallel \triangle BAD$$

$$\checkmark \frac{BK}{BA} = \frac{BC}{BD} = 3$$

$$\checkmark BD = BC + r$$

$$\checkmark BC = \frac{1}{2}r$$

$$\checkmark \cos 2x = \frac{1}{2}$$

$$\checkmark 2x = 60^\circ$$

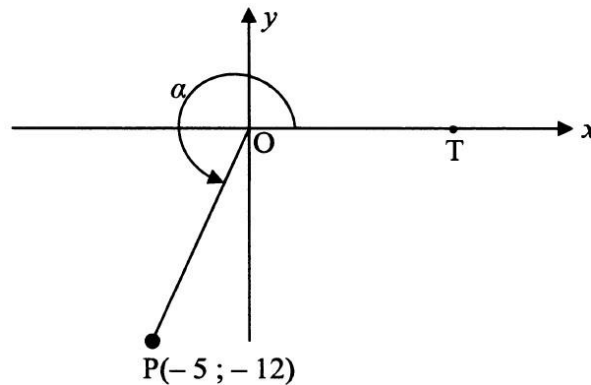
$$\checkmark 30^\circ$$

(8)

[11]

QUESTION 10

In the diagram below, reflex $\hat{TOP} = \alpha$ and P has coordinates $(-5 ; -12)$.



Determine the value of each of the following trigonometric ratios WITHOUT using a calculator:

10.1 $\cos \alpha$ (3)

10.2 $\tan (180^\circ - \alpha)$ (2)

10.3 $\sin (30^\circ - \alpha)$ (3)
[8]

QUESTION 11

11.1 Prove the following identity: $\frac{\cos^2(90^\circ + \theta)}{\cos(-\theta) + \sin(90^\circ - \theta) \cos \theta} = \frac{1}{\cos \theta} - 1$ (6)

11.2 Determine the general solution of: $\tan x \sin x + \cos x \tan x = 0$. (7)

11.3 Consider the following expression: $2 \sin^2 3x - \sin^2 x - \cos^2 x$

11.3.1 Simplify the expression to a single trigonometric ratio of x . (3)

11.3.2 Write down the maximum value of the expression. (1)

11.4 It is given that $p = \cos \alpha + \sin \alpha$ and $q = \cos \alpha - \sin \alpha$

11.4.1 Determine the following trigonometric ratios in terms of p and/or q :

(a) $\cos 2\alpha$ (3)

(b) $\tan \alpha$ (4)

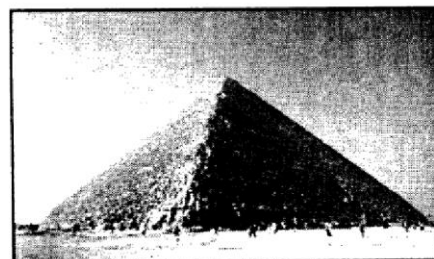
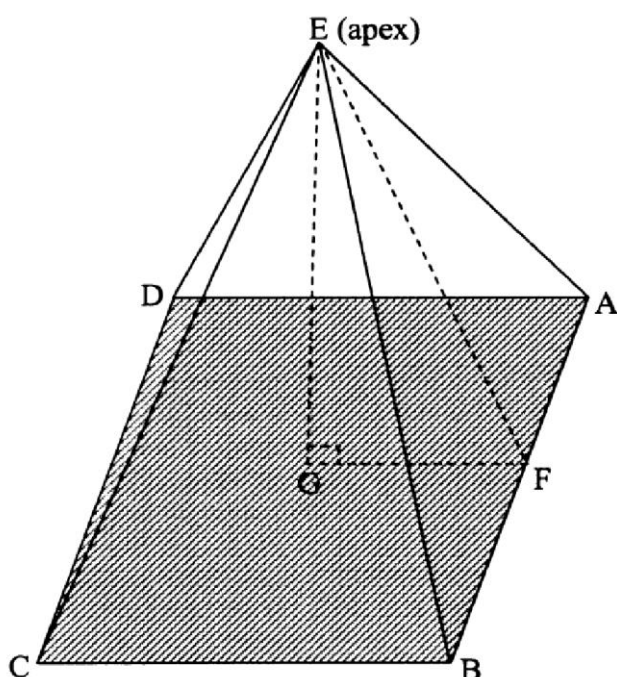
11.4.2 Simplify $\frac{p}{2q} - \frac{q}{2p}$ to a single trigonometric ratio of α . (6)
[30]

QUESTION 12

- 12.1 Draw the graphs of $f(x) = \tan x + 1$ and $g(x) = \cos 2x$ for $x \in [-180^\circ; 180^\circ]$ on the same system of axes provided on DIAGRAM SHEET 2. Clearly show all intercepts with the axes, turning points and asymptotes. (6)
- 12.2 Write down the period of g . (1)
- 12.3 If $h(x) = -\cos 2(x + 10^\circ)$, describe fully, in words, the transformation from g to h . (2)
- 12.4 For which values of x , where $x > 0$, will $f'(x)g(x) > 0$? (4)
- [13]**

QUESTION 13

The Great Pyramid at Giza in Egypt was built around 2 500 BC. The pyramid has a square base (ABCD) with sides 232,6 metres long. The distance from each corner of the base to the apex (E) was originally 221,2 metres.



Great Pyramid at Giza in Egypt

- 13.1 Calculate the size of the angle at the apex of a face of the pyramid (for example $\hat{CEB}$). (3)
- 13.2 Calculate the angle each face makes with the base (for example $\hat{EFG}$, where $EF \perp AB$ in $\triangle AEB$). (6)
- [9]**

QUESTION/VR4AG 10

10.1	$OP^2 = 25 + 144 = 169$ $OP = 13$ $\cos \alpha = -\frac{5}{13}$ $r^2 = x^2 + y^2$ $25 + 144 = 169$ $r = 13$ $\cos \alpha = -\frac{5}{13}$	$\checkmark OP^2 = 25 + 144$ $\checkmark OP = 13$ $\checkmark$ answer (3) OR $\checkmark r^2 = 25 + 144$ $\checkmark r = 13$ $\checkmark$ answer (3)
10.2	$\tan(180^\circ - \alpha)$ $= -\tan \alpha$ $= -\frac{12}{5}$	$\checkmark -\tan \alpha$ $\checkmark$ answer (2)
10.3	$\sin(30^\circ - \alpha)$ $= \sin 30^\circ \cos \alpha - \cos 30^\circ \sin \alpha$ $= \left(\frac{1}{2}\right)\left(\frac{-5}{13}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-12}{13}\right)$ $= \frac{-5 + 12\sqrt{3}}{26}$	$\checkmark$ expansion $\checkmark \left(\frac{1}{2}\right)\left(\frac{-5}{13}\right)$ $\checkmark \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-12}{13}\right)$ (3) [8]

QUESTION/VR4AG 11

11.1	$LHS = \frac{\cos^2(90^\circ + \theta)}{\cos(-\theta) + \sin(90^\circ - \theta) \cdot \cos \theta}$ $= \frac{(-\sin \theta)^2}{\cos \theta + \cos \theta \cdot \cos \theta}$ $= \frac{1 - \cos^2 \theta}{\cos \theta + \cos^2 \theta}$ $= \frac{(1 - \cos \theta)(1 + \cos \theta)}{\cos \theta(1 + \cos \theta)}$ $= \frac{1 - \cos \theta}{\cos \theta}$ $= \frac{1}{\cos \theta} - 1$ $= RHS$	$\checkmark \cos^2(90^\circ + \theta) = \sin^2 \theta$ $\checkmark \sin(90^\circ - \theta) = \cos \theta$ $\checkmark \cos(-\theta) = \cos \theta$ $\checkmark 1 - \cos^2 \theta$ $\checkmark$ factors $\checkmark \frac{1 - \cos \theta}{\cos \theta}$ (6)
11.2	$\tan x \cdot \sin x + \cos x \cdot \tan x = 0$ $\tan x(\sin x + \cos x) = 0$ $\tan x = 0$ or $\sin x + \cos x = 0$ $\sin x = -\cos x$ $\tan x = -1$ $x = 0^\circ + k \cdot 180^\circ; k \in Z$ or $x = 135^\circ + k \cdot 180^\circ; k \in Z$ OR $\frac{\sin x}{\cos x} \cdot \sin x + \cos x \cdot \frac{\sin x}{\cos x} = 0$ $\frac{\sin^2 x}{\cos x} + \frac{\cos x \cdot \sin x}{\cos x} = 0$ $\sin^2 x + \cos x \cdot \sin x = 0$ $\sin x(\sin x + \cos x) = 0$ $\sin x + \cos x = 0$ $\sin x = 0$ or $\tan x = -1$ $x = 0^\circ + k \cdot 180^\circ; k \in Z$ or $x = 135^\circ + k \cdot 180^\circ; k \in Z$ OR $\tan x \cdot \sin x + \cos x \cdot \tan x = 0$ ($\cos x \neq 0$) $\tan^2 x + \tan x = 0$ $\tan x(\tan x + 1) = 0$ $\tan x = 0$ or $\tan x + 1 = 0$ $\tan x = -1$ $x = 0^\circ + k \cdot 180^\circ; k \in Z$ or $x = 135^\circ + k \cdot 180^\circ; k \in Z$ $(\sin x + \cos x)^2 = 0$ $1 + \sin 2x = 0$ $\sin 2x = -1$ ✓ $x = 135^\circ + k \cdot 180^\circ$	$\checkmark$ factorising $\checkmark \tan x = 0$ and $\sin x + \cos x = 0$ $\checkmark \tan x = -1$ $\checkmark x = 0^\circ$ $\checkmark x = 135^\circ$ or -45° $\checkmark k \cdot 180^\circ$ $\checkmark k \in Z$ (7) $\checkmark$ factorising $\checkmark \sin x = 0$ and $\checkmark \tan x = -1$ $\checkmark x = 0^\circ$ $\checkmark x = 135^\circ$ or -45° $\checkmark k \cdot 180^\circ$ $\checkmark k \in Z$ (7) $\checkmark$ factorising $\checkmark \tan x = 0$ and $\tan x + 1 = 0$ $\checkmark \tan x = -1$ $\checkmark x = 0^\circ$ $\checkmark x = 135^\circ$ or -45° $\checkmark k \cdot 180^\circ$ $\checkmark k \in Z$ (7)

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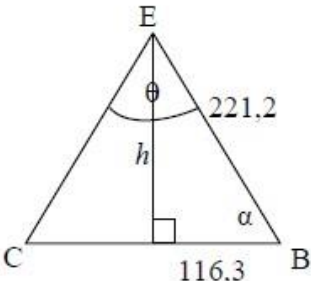
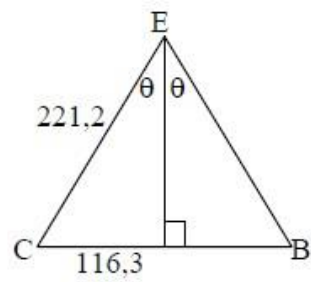
		✓ expansion ✓ $\frac{p-q}{2}$ ✓ $\frac{p+q}{2}$ (3)
11.4.1 (b)	$p+q = 2\cos\alpha \quad \therefore \cos\alpha = \frac{p+q}{2}$ $p-q = 2\sin\alpha \quad \therefore \sin\alpha = \frac{p-q}{2}$ $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ $= \frac{2\sin\alpha}{2\cos\alpha}$ $= \frac{p-q}{p+q}$ OR $\cos\alpha + \sin\alpha = p$ $\cos\alpha - \sin\alpha = q$ $\Rightarrow 2\cos\alpha = p+q$ $\cos\alpha = \frac{p+q}{2}$ $y^2 = 2^2 - (p+q)^2$ $y = \sqrt{4 - (p+q)^2}$ $\therefore \tan\alpha = \frac{\sqrt{4 - (p+q)^2}}{p+q}$ OR $\cos\alpha + \sin\alpha = p$ $\cos\alpha - \sin\alpha = q$ $\Rightarrow 2\sin\alpha = p-q$ $\sin\alpha = \frac{p-q}{2}$ $x^2 = 2^2 - (p-q)^2$ $x = \sqrt{4 - (p-q)^2}$ $\therefore \tan\alpha = \frac{p-q}{\sqrt{4 - (p-q)^2}}$ OR	✓ $\frac{p+q}{2}$ ✓ $\frac{p-q}{2}$ ✓ identity ✓ answer (4) ✓ $2\cos\alpha = p+q$ ✓ sketch ✓ $y = \sqrt{4 - (p+q)^2}$ ✓ answer (4) ✓ $2\sin\alpha = p-q$ ✓ sketch ✓ $x = \sqrt{4 - (p-q)^2}$ ✓ answer (4)

	$\cos 2\alpha = 1 - 2 \sin^2 \alpha$ $\sin^2 \alpha = \frac{1 - pq}{2}$ $\cos 2\alpha = 2 \cos^2 \alpha - 1$ $\cos^2 \alpha = \frac{pq + 1}{2}$ $(\tan \alpha)^2 = \left(\frac{\sin \alpha}{\cos \alpha} \right)^2$ $\tan^2 \alpha = \frac{1 - pq}{1 + pq}$ $\therefore \tan \alpha = \sqrt{\frac{1 - pq}{1 + pq}}$	$\checkmark \sin^2 \alpha = \frac{1 - pq}{2}$ $\checkmark \cos^2 \alpha = \frac{pq + 1}{2}$ $\checkmark (\tan \alpha)^2 = \left(\frac{\sin \alpha}{\cos \alpha} \right)^2$ $\checkmark \tan \alpha = \sqrt{\frac{1 - pq}{1 + pq}}$ (4)
11.4.2	$\frac{p}{2q} - \frac{q}{2p}$ $= \frac{p^2 - q^2}{2pq}$ $= \frac{(p + q)(p - q)}{2pq}$ $= \frac{(2 \cos \alpha)(2 \sin \alpha)}{2 \cos 2\alpha}$ $= \frac{4 \sin \alpha \cos \alpha}{2 \cos 2\alpha}$ $= \frac{2 \sin 2\alpha}{2 \cos 2\alpha}$ $= \tan 2\alpha$ OR $\frac{p}{2q} - \frac{q}{2p}$ $= \frac{\cos \alpha + \sin \alpha}{2(\cos \alpha - \sin \alpha)} - \frac{\cos \alpha - \sin \alpha}{2(\cos \alpha + \sin \alpha)}$ $= \frac{(\cos \alpha + \sin \alpha)^2 - (\cos \alpha - \sin \alpha)^2}{2(\cos \alpha - \sin \alpha)(\cos \alpha + \sin \alpha)}$ $= \frac{\cos^2 \alpha + 2 \sin \alpha \cos \alpha + \sin^2 \alpha - (\cos^2 \alpha - 2 \sin \alpha \cos \alpha + \sin^2 \alpha)}{2(\cos^2 \alpha - \sin^2 \alpha)}$ $= \frac{4 \sin \alpha \cos \alpha}{2 \cos 2\alpha}$ $= \frac{2 \sin 2\alpha}{2 \cos 2\alpha}$ $= \tan 2\alpha$	$\checkmark \frac{p^2 - q^2}{2pq}$ $\checkmark \text{ factorising}$ $\checkmark \text{ substituting}$ $\checkmark \text{ from 11.4.1(a)}$ $\checkmark \text{ and 11.4.1(b)}$ $\checkmark 2 \sin 2\alpha$ $\checkmark \tan 2\alpha$ (6) OR $\checkmark \text{ substitution}$ $\checkmark \text{ single fraction}$ $\checkmark 4 \sin \alpha \cos \alpha$ $\checkmark 2 \cos 2\alpha$ $\checkmark 2 \sin 2\alpha$ $\checkmark \tan 2\alpha$ (6) [30]

QUESTION/VRAAG 12

12.1		$y = \tan x + 1$ ✓ asymptotes and shape for whole domain ✓ y intercept ✓ x intercepts $y = \cos 2x$ ✓ x intercepts ✓ y intercept/TP ✓ minimum values (6)
12.2	Period of g is 180° .	✓ 180° (1)
12.3	Reflected about the x -axis and then translated by 10° to the left/ <i>refleksie om die x-as en dan 'n translasie van 10° links.</i> OR Translated by 10° to the left and then reflected about the x -axis/ <i>Translasie van 10° links en dan 'n refleksie om die x-as.</i>	✓ reflected about x -axis/ <i>refleksie om x-as</i> ✓ 10° to the left 10° na links (2) ✓ 10° to the left 10° na links ✓ reflected about x -axis/ <i>refleksie om x-as</i> (2)
12.4	f is always increasing $\therefore f'(x) > 0$ always $\therefore g(x) > 0$ $\therefore 0^\circ < x < 45^\circ$ or $135^\circ < x \leq 180^\circ$ OR $\therefore x \in (0^\circ; 45^\circ)$ or $(135^\circ; 180^\circ]$	✓ critical values 0° and 45° ✓ inequality ✓ critical values 135° and 180° ✓ inequality (4) [13]

QUESTION/VRAAG 13

13.1	In $\triangle CEB$: $BC^2 = EC^2 + EB^2 - 2(EC)(EB)\cos\hat{C}EB$ $(232,6)^2 = (221,2)^2 + (221,2)^2 - 2(221,2)(221,2)\cos\hat{C}EB$ $\cos\hat{C}EB = \frac{2(221,2)^2 - (232,6)^2}{2(221,2)^2}$ $= 0,447.....$ $\hat{C}EB = 63,44^\circ$ OR $\cos \alpha = \frac{116,3}{221,2}$ $\alpha = 58,28^\circ$ $\theta = 180^\circ - 2(58,28^\circ) = 63,44^\circ$  OR $\sin \theta = \frac{116,3}{221,2}$ $\theta = 31,72^\circ$ $\hat{AEB} = 2\theta = 2(31,72^\circ) = 63,44^\circ$ 	✓ substitution into correct formula ✓ 0,447.... ✓ $63,44^\circ$ (3) ✓ substitution into correct definition ✓ $\alpha = 58,28^\circ$ ✓ $63,44^\circ$ (3) ✓ substitution into correct formula ✓ $\theta = 31,72^\circ$ ✓ $63,44^\circ$ (3)
13.2	$EF^2 = EB^2 - BF^2$ $= (221,2)^2 - (116,3)^2$ $= 35403,75$ $EF = 188,16\text{ m}$ $\cos \hat{E}FG = \frac{GF}{EF}$ $= \frac{116,3}{188,16}$ $= 0,618....$ $\hat{E}FG = 51,82^\circ$ OR	✓ using Pythagoras correctly ✓ $BF = 116,3$ ✓ 188,16 ✓ using $\cos \hat{E}FG$ ✓ $\frac{116,3}{188,16}$ ✓ $51,82^\circ$ (6)

$$\sin \alpha = \frac{h}{221,2}$$

$$h = 221,2 \sin 58,28^\circ$$

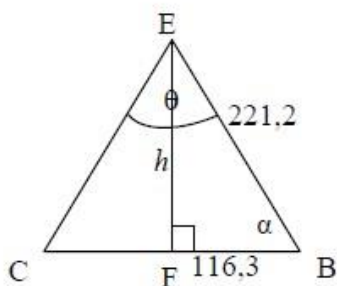
$$= 188,158...$$

In $\triangle EFG$:

$$\cos \hat{EFG} = \frac{116,3}{188,158...}$$

$$= 0,61809...$$

$$\hat{EFG} = 51,82^\circ$$



$$\checkmark \sin \alpha = \frac{h}{221,2}$$

$\checkmark h$ subject

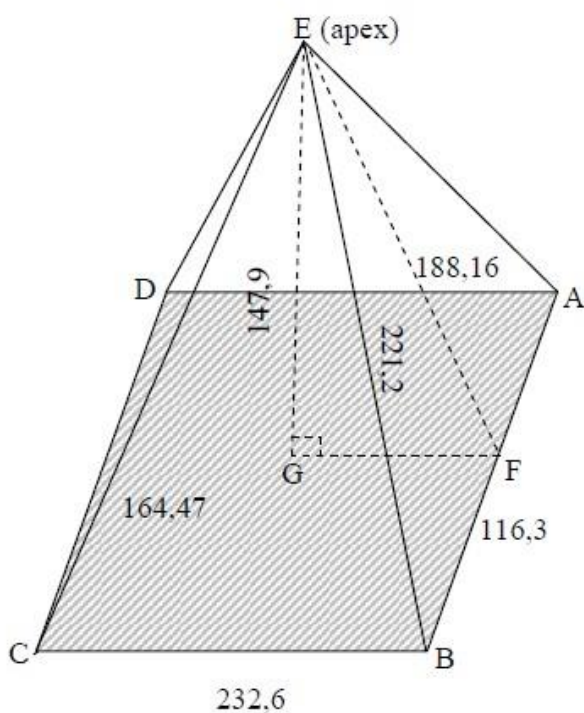
$$\checkmark h = 188,158...$$

$\checkmark$ using $\cos \hat{EFG}$

$$\checkmark \frac{116,3}{188,16}$$

$$\checkmark 51,82^\circ$$

(6)



[9]

QUESTION 8

8.1 If $\sin A = \frac{3}{5}$ and $\cos A < 0$, determine, WITHOUT using a calculator, the value of:

8.1.1 $\sin(-A)$ (2)

8.1.2 $\tan A$ (3)

8.2 If $\cos 34^\circ = p$, WITHOUT using a calculator, write down the following in terms of p :

8.2.1 $\cos 214^\circ$ (2)

8.2.2 $\cos 68^\circ$ (2)

8.2.3 $\tan 56^\circ$ (4)

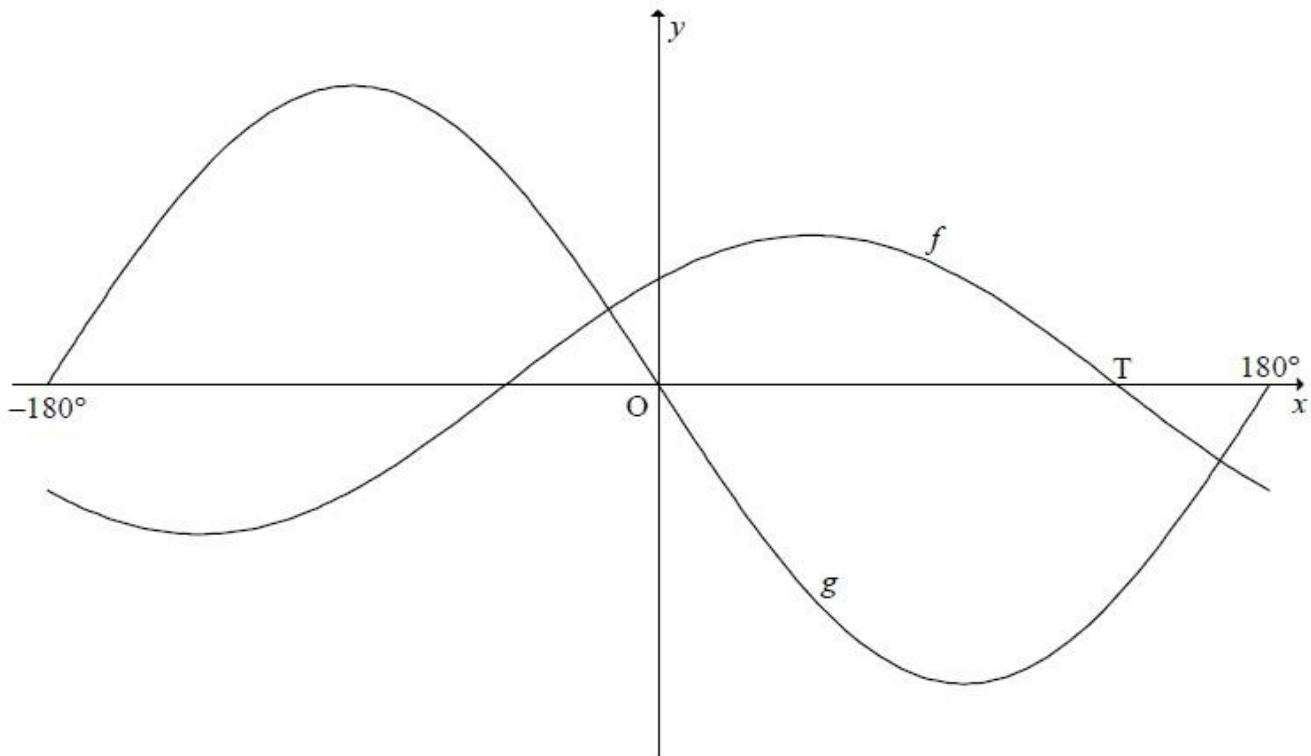
8.3 WITHOUT using a calculator, determine the value of the following expression:

$$\cos 350^\circ \sin 40^\circ - \cos 440^\circ \cos 40^\circ$$

(5)
[18]

QUESTION 9

The graphs of $f(x) = \cos(x - 45^\circ)$ and $g(x) = -2 \sin x$ are drawn below for $x \in [-180^\circ; 180^\circ]$. The point T is an x-intercept of f as indicated on the diagram.

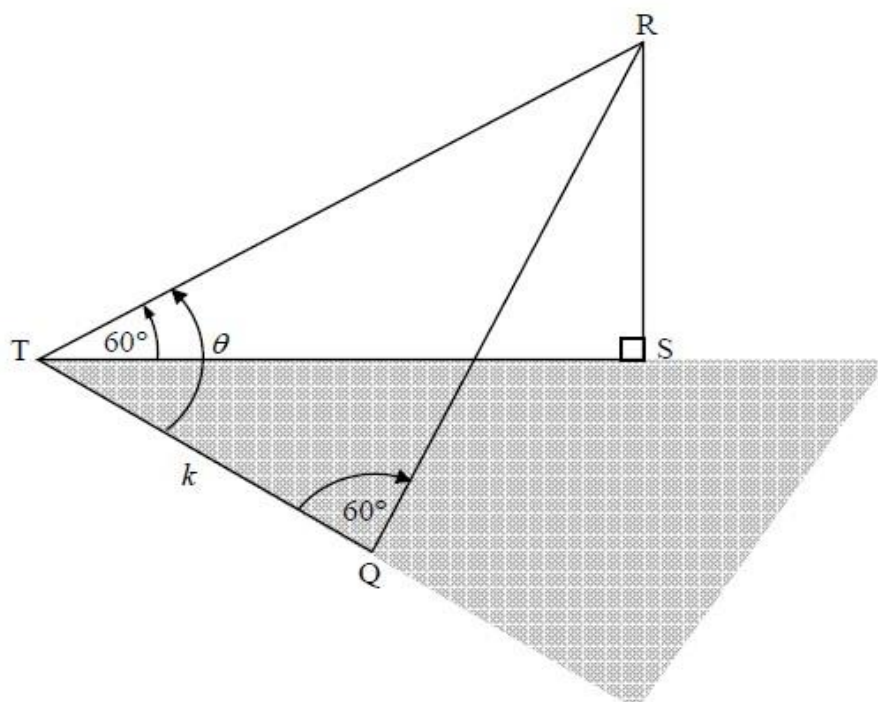


- 9.1 Show that $\cos(x - 45^\circ) = -2 \sin x$ can be written as $\tan x = -0,2612$. (4)
- 9.2 Solve the equation: $\cos(x - 45^\circ) = -2 \sin x$ for $x \in [-180^\circ; 180^\circ]$. (3)
- 9.3 Write down the coordinates of point T. (2)
- 9.4 Write down the interval for which $f(x) \geq g(x)$. (2)
- 9.5 Write down the interval for which both f and g are strictly increasing. (3)
- 9.6 The graph h is obtained when the graph f is shifted 45° to the right. Write down the equation of h in its simplest form. (2)

[16]

QUESTION 10

In the diagram below, RS is the height of a vertical tower. T and Q are two points in the same horizontal plane as the foot S of the tower. From point T the angle of elevation to the top of the tower is 60° . $\hat{RTQ} = \theta$, $\hat{RQT} = 60^\circ$ and $TQ = k$ metres.



10.1 Express TR in terms of θ and k . (3)

10.2 Show that $RS = \frac{3k}{2(\sqrt{3} \cos \theta + \sin \theta)}$. (7)
[10]

QUESTION 11

11.1 Consider the function: $f(x) = 3 - 2\sin^2 x$

11.1.1 Determine the range of f . (4)

11.1.2 For which value(s) x , $x \in [-180^\circ; 180^\circ]$, will f have a minimum value? (3)

11.2 11.2.1 Show that $1 - \cos 2Q = 2\sin^2 Q$. (1)

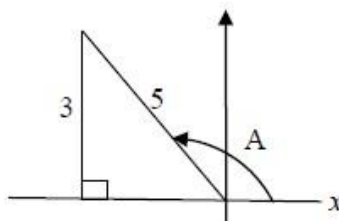
11.2.2 Given: $\hat{P} + \hat{Q} + \hat{R} = 180^\circ$

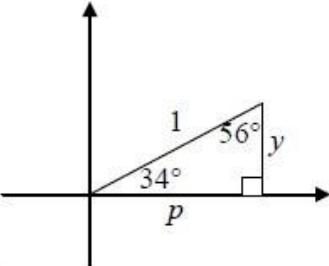
(a) Show that $\sin 2R = -\sin(2P + 2Q)$. (3)

(b) Hence, show that
 $\sin 2P + \sin 2Q + \sin 2R = 4 \sin P \sin Q \sin R$. (7)
[18]

QUESTION/VR4AG 8

8.1.1	$\sin A = \frac{3}{5}$ (given) $\sin(-A)$ $= -\sin A$ $= -\frac{3}{5}$	✓ sin A ✓ value (2)
8.1.2	$\sin^2 A + \cos^2 A = 1$ $\cos^2 A = 1 - \frac{9}{25} = \frac{16}{25}$ $\cos A = -\frac{4}{5}$ $\tan A = \frac{\sin A}{\cos A}$ $= \frac{3}{5} \times -\frac{5}{4}$ $= -\frac{3}{4}$ OR $x = -4$ $\tan A = -\frac{3}{4}$	✓ $\cos^2 A = \frac{16}{25}$ ✓ $\cos A = -\frac{4}{5}$ ✓ ratio ✓ sketch in correct quadrant ✓ $x = -4$ ✓ ratio (3)
8.2.1	$\cos 214^\circ$ $= \cos (180^\circ + 34^\circ)$ $= -\cos 34^\circ$ $= -p$	✓ $-\cos 34^\circ$ ✓ $-p$ (2)
8.2.2	$\cos 68^\circ$ $= \cos [2(34^\circ)]$ $= 2\cos^2 34^\circ - 1$ $= 2p^2 - 1$	✓ $\cos [2(34^\circ)]$ ✓ $2p^2 - 1$ (2)



8.2.3	$\begin{aligned}\tan 56^\circ &= \frac{\sin 56^\circ}{\cos 56^\circ} \\ &= \frac{\cos 34^\circ}{\sin 34^\circ} \\ &= \frac{\cos 34^\circ}{\sqrt{1 - \cos^2 34^\circ}} \\ &= \frac{p}{\sqrt{1 - p^2}}\end{aligned}$ OR $\begin{aligned}y^2 &= 1 - p^2 \\ y &= \sqrt{1 - p^2} \\ \therefore \tan 56^\circ &= \frac{p}{\sqrt{1 - p^2}}\end{aligned}$ 	✓ identity ✓ co-functions ✓ $\sqrt{1 - \cos^2 34^\circ}$ ✓ answer (4) ✓ sketch with 34° ✓ $y = \sqrt{1 - p^2}$ ✓ $90^\circ - 34^\circ = 56^\circ$ ✓ answer (4)
8.3	$\begin{aligned}\cos 350^\circ \sin 40^\circ - \cos 440^\circ \cos 40^\circ \\ &= \cos 10^\circ \sin 40^\circ - \cos 80^\circ \cos 40^\circ \\ &= \cos 10^\circ \sin 40^\circ - \sin 10^\circ \cos 40^\circ \\ &= \sin 40^\circ \cos 10^\circ - \cos 40^\circ \sin 10^\circ \\ &= \sin(40^\circ - 10^\circ) \\ &= \sin 30^\circ \\ &= \frac{1}{2}\end{aligned}$ OR $\begin{aligned}\cos 350^\circ \sin 40^\circ - \cos 440^\circ \cos 40^\circ \\ &= \cos 10^\circ \sin 40^\circ - \cos 80^\circ \cos 40^\circ \\ &= \cos 10^\circ \cos 50^\circ - \sin 10^\circ \sin 50^\circ \\ &= \cos(10^\circ + 50^\circ) \\ &= \cos 60^\circ \\ &= \frac{1}{2}\end{aligned}$ NOTE: There are many solutions.	✓ $\cos 10^\circ$ ✓ $\cos 80^\circ$ ✓ $\sin 10^\circ$ ✓ $\sin 30^\circ$ ✓ answer (5) ✓ $\cos 10^\circ$ ✓ $\cos 80^\circ$ ✓ $\cos 50^\circ$ and $\sin 50^\circ$ ✓ $\cos 60^\circ$ ✓ answer (5) [18]

QUESTION/VRAAG 9

9.1	$\cos(x - 45^\circ) = -2 \sin x$ $\cos x \cos 45^\circ + \sin x \sin 45^\circ = -2 \sin x$ $\frac{\sqrt{2}}{2} \cos x + \frac{\sqrt{2}}{2} \sin x = -2 \sin x$ $\sqrt{2} \cos x = (-4 - \sqrt{2}) \sin x$ $\sqrt{2} = \frac{(-4 - \sqrt{2}) \sin x}{\cos x}$ $\tan x = \frac{\sqrt{2}}{-4 - \sqrt{2}} = -0,2612$	✓ expansion ✓ $\frac{\sqrt{2}}{2}$ ✓ simplification ✓ dividing by $\cos x$ (4)
9.2	$\tan x = -0,2612...$ $x = 165,36^\circ + 180^\circ k; k \in \mathbb{Z}$ $x = -14,64^\circ \text{ or } 165,36^\circ$	✓ general solution ✓ ✓ values of x (3)
9.3	T (135° ; 0)	✓ x value ✓ y value (2)
9.4	$f(x) \geq g(x)$ $-14,64^\circ \leq x \leq 165,36^\circ \quad \text{OR} \quad x \in [-14,64^\circ ; 165,36^\circ]$	✓ extreme values ✓ notation (2)
9.5	$-135^\circ < x < -90^\circ \quad \text{OR} \quad x \in (-135^\circ ; -90^\circ)$	✓ ✓ extreme values ✓ notation (3)
9.6	$h(x) = \cos(x - 45^\circ - 45^\circ)$ $= \cos(x - 90^\circ)$ $= \sin x$ Answer only: full marks	✓ $\cos(x - 90^\circ)$ ✓ $\sin x$ (2)

[16]

QUESTION/VR4AG 10

10.1	In $\triangle TRQ$: $\frac{TR}{\sin \hat{RQT}} = \frac{TQ}{\sin \hat{QRT}}$ $\frac{TR}{\sin 60^\circ} = \frac{k}{\sin[180^\circ - (\theta + 60^\circ)]}$ $TR = \frac{k \sin 60^\circ}{\sin[180^\circ - (\theta + 60^\circ)]} \quad \text{or} \quad TR = \frac{k \sin 60^\circ}{\sin(120^\circ - \theta)}$	✓ using sine rule ✓ correct substitution ✓ rewrite TR as subject (3)
10.2	In $\triangle TRS$: $\frac{RS}{TR} = \sin \hat{RTS}$ $RS = TR \cdot \sin 60^\circ$ $= \frac{k \sin 60^\circ}{\sin[180^\circ - (\theta + 60^\circ)]} \cdot \sin 60^\circ$ $= \frac{k \sin 60^\circ}{\sin(120^\circ - \theta)} \cdot \sin 60^\circ$ $= \frac{k\left(\frac{\sqrt{3}}{2}\right)^2}{\sin 120^\circ \cos \theta - \cos 120^\circ \sin \theta}$ $= \frac{3k}{4\left(\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta\right)}$ $= \frac{3k}{2(\sqrt{3} \cos \theta + \sin \theta)}$	✓ using sine ratio ✓ substitution of TR ✓ simplification ✓ $k\left(\frac{\sqrt{3}}{2}\right)^2$ or $\frac{3k}{4}$ ✓ expansion of denominator ✓ value of $\sin 120^\circ$ ✓ value of $\cos 120^\circ$ (7) [10]

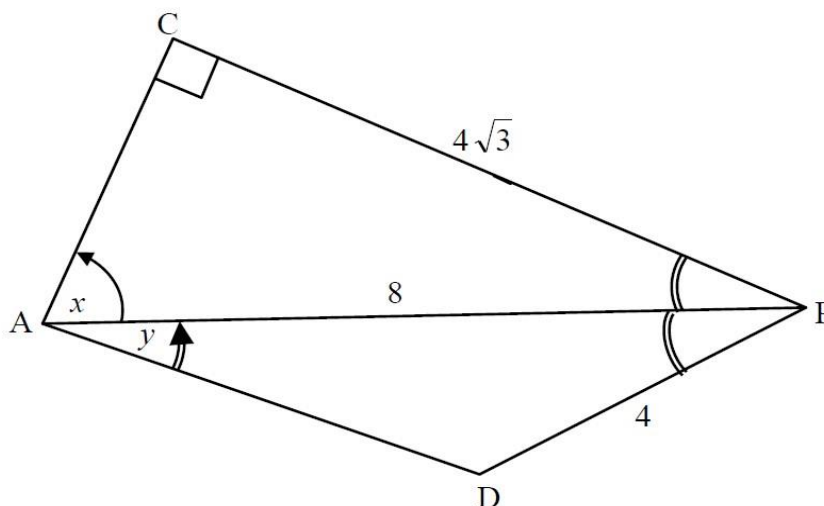
QUESTION/VRAAG 11

11.1.1	$f(x) = y = 3 - 2 \sin^2 x$ $0 \leq \sin^2 x \leq 1$ $-2 \leq -2 \sin^2 x \leq 0$ $1 \leq 3 - 2 \sin^2 x \leq 3$ $1 \leq y \leq 3$ OR $y \in [1 ; 3]$ OR $f(x) = y = 3 - 2 \sin^2 x$ $= 2 + (1 - 2 \sin^2 x)$ $= \cos 2x + 2$ $-1 \leq \cos 2x \leq 1$ $1 \leq \cos 2x + 2 \leq 3$ $1 \leq y \leq 3$ OR $y \in [1 ; 3]$ OR $f(x) = y = 3 - 2 \sin^2 x$ $= 3 - 2(1 - \cos^2 x)$ $= 1 + 2 \cos^2 x$ $0 \leq 2 \cos^2 x \leq 2$ $1 \leq 1 + 2 \cos^2 x \leq 3$ $1 \leq y \leq 3$ OR $y \in [1 ; 3]$	✓ ✓ $-2 \leq -2 \sin^2 x \leq 0$ ✓ ✓ $1 \leq 3 - 2 \sin^2 x \leq 3$ (4) ✓ rewriting ✓ $\cos 2x + 2$ ✓ $-1 \leq \cos 2x \leq 1$ ✓ $1 \leq \cos 2x + 2 \leq 3$ (4) ✓ rewriting ✓ $1 + 2 \cos^2 x$ ✓ $0 \leq 2 \cos^2 x \leq 2$ ✓ $1 \leq 1 + 2 \cos^2 x \leq 3$ (4)
11.1.2	f has a minimum when $\sin^2 x = 1$ $\therefore \sin x = \pm 1$ $\therefore x = 90^\circ$ or -90° OR f has a minimum when $\cos 2x = -1$ $\therefore 2x = 180^\circ$ or -180° $\therefore x = 90^\circ$ or -90°	✓ $\sin^2 x = 1$ ✓ 90° ✓ -90° (3) ✓ $\cos 2x = -1$ ✓ 90° ✓ -90° (3)
11.2.1	$LHS = 1 - \cos 2Q$ $= 1 - (1 - 2 \sin^2 Q)$ $= 2 \sin^2 Q$ $= RHS$	✓ identity (1)
11.2.2(a)	$LHS = \sin 2R$ $= \sin 2[180^\circ - (P + Q)]$ $= \sin[360^\circ - 2(P + Q)]$ $= -\sin 2(P + Q)$ $= -\sin(2P + 2Q)$ $= RHS$	✓ $R = 180^\circ - (P + Q)$ ✓ $360^\circ - 2(P + Q)$ ✓ $-2(P + Q)$ (3)

11.2.2(b)	$ \begin{aligned} LHS &= \sin 2P + \sin 2Q + \sin 2R \\ &= \sin 2P + \sin 2Q - \sin(2P + 2Q) \\ &= \sin 2P + \sin 2Q - [\sin 2P \cos 2Q + \cos 2P \sin 2Q] \\ &= \sin 2P + \sin 2Q - \sin 2P \cos 2Q - \cos 2P \sin 2Q \\ &= \sin 2P(1 - \cos 2Q) + \sin 2Q(1 - \cos 2P) \\ &= \sin 2P(2 \sin^2 Q) + \sin 2Q(2 \sin^2 P) \\ &= 2 \sin P \cos P \cdot 2 \sin^2 Q + 2 \sin Q \cos Q \cdot 2 \sin^2 P \\ &= 4 \sin P \sin Q (\sin Q \cos P + \cos Q \sin P) \\ &= 4 \sin P \sin Q (\sin(Q + P)) \\ &= 4 \sin P \sin Q (\sin[180^\circ - (Q + P)]) \\ &= 4 \sin P \sin Q \sin R \\ &= RHS \end{aligned} $	✓ substitution ✓ expansion ✓ factorising ✓ substitution ✓ identities ✓ factorising ✓ $[180^\circ - (Q + P)]$ (7) [18]
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QUESTION 5

In the figure below, $\triangle ACP$ and $\triangle ADP$ are triangles with $\hat{C} = 90^\circ$, $CP = 4\sqrt{3}$, $AP = 8$ and $DP = 4$. PA bisects $\hat{DPC}$. Let $\hat{CAP} = x$ and $\hat{DAP} = y$.



5.1 Show, by calculation, that $x = 60^\circ$. (2)

5.2 Calculate the length of AD . (4)

5.3 Determine y . (3)

[9]

QUESTION 6

6.1 Prove the identity: $\cos^2(180^\circ + x) + \tan(x - 180^\circ)\sin(720^\circ - x)\cos x = \cos 2x$ (5)

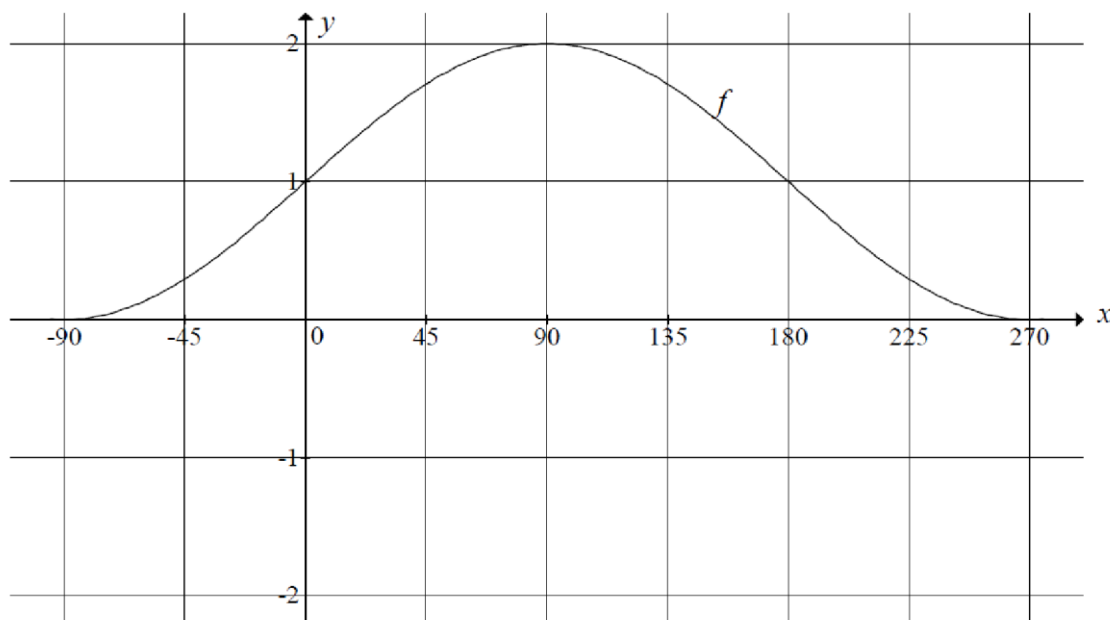
6.2 Use $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ to derive the formula for $\sin(\alpha - \beta)$. (3)

6.3 If $\sin 76^\circ = x$ and $\cos 76^\circ = y$, show that $x^2 - y^2 = \sin 62^\circ$. (4)

[12]

QUESTION 7

In the diagram below, the graph of $f(x) = \sin x + 1$ is drawn for $-90^\circ \leq x \leq 270^\circ$.



- 7.1 Write down the range of f . (2)
- 7.2 Show that $\sin x + 1 = \cos 2x$ can be rewritten as $(2 \sin x + 1) \sin x = 0$. (2)
- 7.3 Hence, or otherwise, determine the general solution of $\sin x + 1 = \cos 2x$. (4)
- 7.4 Use the grid on DIAGRAM SHEET 2 to draw the graph of $g(x) = \cos 2x$ for $-90^\circ \leq x \leq 270^\circ$. (3)
- 7.5 Determine the value(s) of x for which $f(x + 30^\circ) = g(x + 30^\circ)$ in the interval $-90^\circ \leq x \leq 270^\circ$. (3)
- 7.6 Consider the following geometric series:
- $$1 + 2 \cos 2x + 4 \cos^2 2x + \dots$$
- Use the graph of g to determine the value(s) of x in the interval $0^\circ \leq x \leq 90^\circ$ for which this series will converge. (5)

[19]

5.1	$\sin \hat{C}AP = \frac{CP}{AP}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60^\circ$ OR/OF $\frac{\sin 90^\circ}{8} = \frac{\sin x}{4\sqrt{3}}$ $\sin x = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $x = 60^\circ$	✓ correct sine ratio/ korrekte sin-verh ✓ $\frac{\sqrt{3}}{2}$ (2) ✓ correct sine ratio/ korrekte sin-verh ✓ $\frac{\sqrt{3}}{2}$ (2)
5.2	$\hat{C}PA = \hat{D}PA = 30^\circ \quad (\text{AP bisects } \hat{D}PC)$ $AD^2 = AP^2 + DP^2 - 2 \cdot AP \cdot DP \cdot \cos \hat{A}PD$ $= 8^2 + 4^2 - 2(8)(4) \cos 30^\circ$ $= 8^2 + 4^2 - 2(8)(4) \left(\frac{\sqrt{3}}{2}\right)$ $= 24,57...$ $AD = 4,96$	✓ $\hat{D}PA = 30^\circ$ ✓ correct subst into cosine rule/ korrekte subst in cos-reël ✓ 24,57... ✓ 4,96 (4)
5.3	$\frac{\sin \hat{D}AP}{DP} = \frac{\sin \hat{A}PD}{AD}$ $\frac{\sin y}{4} = \frac{\sin 30^\circ}{4,96}$ $\sin y = \frac{4 \sin 30^\circ}{4,96}$ $= 0,403...$ $y = 23,78^\circ$ OR/OF $AD^2 = AP^2 + DP^2 - 2 \cdot AP \cdot DP \cdot \cos \hat{D}AP$ $4^2 = 8^2 + (4,96)^2 - 2(8)(4,96) \cdot \cos y$ $\cos y = \frac{8^2 + (4,96)^2 - 4^2}{2(8)(4,96)}$ $\cos y = 0,9148...$ $y = 23,82^\circ$	✓ correct subst into sine rule/ korrekte subst in sin-reël ✓ sin y subject ✓ 23,78° (3) ✓ correct subst into cosine rule/ korrekte subst in cos-reël ✓ cos y subject ✓ 23,82° (3) [9]

QUESTION/VRAAG 6

6.1	$\cos^2(180^\circ + x) + \tan(x - 180^\circ) \sin(720^\circ - x) \cos x$ $= (-\cos x)^2 + [-(\tan x)](-\sin x)(\cos x)$ $= \cos^2 x + \left(\frac{\sin x}{\cos x}\right)(-\sin x)(\cos x)$ $= \cos^2 x - \sin^2 x$ $= \cos 2x$	$\checkmark (-\cos x)^2 \text{ or } \cos^2 x$ $\checkmark \tan x \text{ or } -(\tan x)$ $\checkmark -\sin x$ $\checkmark \tan x = \frac{\sin x}{\cos x}$ $\checkmark \cos^2 x - \sin^2 x$ (5)
6.2	$\sin(\alpha - \beta)$ $= \cos[90^\circ - (\alpha - \beta)]$ $= \cos[(90^\circ - \alpha) + \beta]$ $= \cos(90^\circ - \alpha) \cos \beta - \sin(90^\circ - \alpha) \sin \beta$ $= \sin \alpha \cos \beta - \cos \alpha \sin \beta$ OR/OF $\sin(\alpha - \beta)$ $= \cos[90^\circ - (\alpha - \beta)]$ $= \cos[(90^\circ + \beta) + (-\alpha)]$ $= \cos(90^\circ + \beta) \cos(-\alpha) - \sin(90^\circ + \beta) \sin(-\alpha)$ $= (-\sin \beta) \cos \alpha - \cos \beta(-\sin \alpha)$ $= \sin \alpha \cos \beta - \cos \alpha \sin \beta$	$\checkmark \text{rewrite as/herskryf}$ $\cos[(90^\circ - \alpha) + \beta]$ $\checkmark \text{expansion/}$ uitbreiding $\checkmark \text{simpl/vereenv}$ (3) $\checkmark \text{rewrite as/herskryf}$ $\cos[(90^\circ + \beta) + (-\alpha)]$ $\checkmark \text{expansion/}$ uitbreiding $\checkmark \text{simpl/vereenv}$ (3)
6.3	$x^2 - y^2$ $= \sin^2 76^\circ - \cos^2 76^\circ$ $= -(\cos^2 76^\circ - \sin^2 76^\circ)$ $= -\cos 2(76^\circ)$ $= -\cos 152^\circ$ $= -(-\cos 28^\circ)$ $= \cos 28^\circ$ $= \cos(90^\circ - 62^\circ)$ $= \sin 62^\circ$ OR/OF $= -\cos(90^\circ + 62^\circ)$ $= -(-\sin 62^\circ)$ $= \sin 62^\circ$ OR/OF $x^2 - y^2$ $= \sin^2 76^\circ - \cos^2 76^\circ$ $= \sin 76^\circ \sin 76^\circ - \cos 76^\circ \cos 76^\circ$ $= \sin 76^\circ \cos 14^\circ - \cos 76^\circ \sin 14^\circ$ $= \sin(76^\circ - 14^\circ)$ $= \sin 62^\circ$ OR/OF $x^2 - y^2$ $= \sin^2 76^\circ - \cos^2 76^\circ$ $= \cos^2 14^\circ - \sin^2 14^\circ$ $= \cos 2(14^\circ)$ $= \cos 28^\circ$ $= \sin 62^\circ$	$\checkmark -(\cos^2 76^\circ - \sin^2 76^\circ)$ $\checkmark \text{recognition of cos}$ double angle $\checkmark -\cos 152^\circ$ $\checkmark \cos 28^\circ$ (4) $\checkmark \cos 14^\circ$ $\checkmark \sin 14^\circ$ $\checkmark \text{recognition of sine}$ compound angle $\checkmark \sin(76^\circ - 14^\circ)$ (4) $\checkmark \cos^2 14^\circ$ $\checkmark \sin^2 14^\circ$ $\checkmark \text{recognition of cos}$ double angle $\checkmark \cos 28^\circ$ (4) [12]

QUESTION/VRAAG 7

7.1	$0 \leq y \leq 2$ or $y \in [0 ; 2]$	✓ critical values/ kritieke waardes ✓ notation/notasie (2)
7.2	$\sin x + 1 = \cos 2x$ $\sin x + 1 = 1 - 2\sin^2 x$ $2\sin^2 x + \sin x = 0$ $\sin x(2\sin x + 1) = 0$	✓ $1 - 2\sin^2 x$ ✓ st form/st vorm (2)
7.3	$\sin x(2\sin x + 1) = 0$ $\sin x = 0$ or $\sin x = -\frac{1}{2}$ $x = 0^\circ + k \cdot 360^\circ$ or $x = 210^\circ + k \cdot 360^\circ$ or $x = 180^\circ + k \cdot 360^\circ$ or $x = 330^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$ OR/OF $x = k \cdot 180^\circ, k \in \mathbb{Z}$	✓ $\sin x = 0$ or $\sin x = -\frac{1}{2}$ ✓ $0^\circ ; 180^\circ$ OR/OF $x = k \cdot 180^\circ$ ✓ $210^\circ ; 330^\circ$ ✓ $k \cdot 360^\circ, k \in \mathbb{Z}$ (4)
7.4		✓ y-intercept/afsnit ✓ x-intercepts/afsnitte ✓ min/max points/ min/maks punte (3)
7.5	$f(x) = g(x)$ at/by: $x = -30^\circ ; 0^\circ ; 180^\circ ; 210^\circ$ $\therefore f(x + 30^\circ) = g(x + 30^\circ)$ at/by: $x = -60^\circ ; -30^\circ ; 150^\circ ; 180^\circ$	✓ $-30^\circ ; 0^\circ ; 180^\circ ; 210^\circ$ ✓ $-60^\circ ; -30^\circ ; 150^\circ ; 180^\circ$ (3)
7.6	Series will converge if/Reeks sal konvergeer as: $-1 < r < 1$ $-1 < 2\cos 2x < 1$ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ $\therefore 30^\circ < x < 60^\circ$ or $x \in (30^\circ ; 60^\circ)$	✓ $-1 < r < 1$ ✓ $r = 2\cos 2x$ ✓ $-\frac{1}{2} < \cos 2x < \frac{1}{2}$ ✓ $30^\circ < x < 60^\circ$ (5) [19]

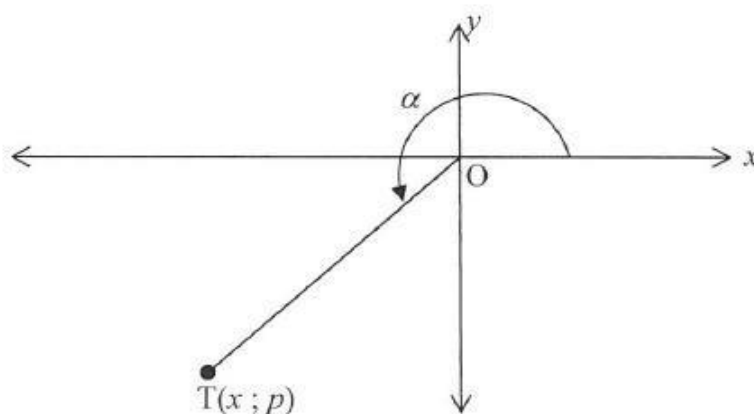
QUESTION 5

5.1 If $x = 3 \sin \theta$ and $y = 3 \cos \theta$, determine the value of $x^2 + y^2$. (3)

5.2 Simplify to a single term:

$$\sin(540^\circ - x) \cdot \sin(-x) - \cos(180^\circ - x) \cdot \sin(90^\circ + x) \quad (6)$$

5.3 In the diagram below, $T(x; p)$ is a point in the third quadrant and it is given that $\sin \alpha = \frac{p}{\sqrt{1+p^2}}$.



5.3.1 Show that $x = -1$. (3)

5.3.2 Write $\cos(180^\circ + \alpha)$ in terms of p in its simplest form. (2)

5.3.3 Show that $\cos 2\alpha$ can be written as $\frac{1-p^2}{1+p^2}$. (3)

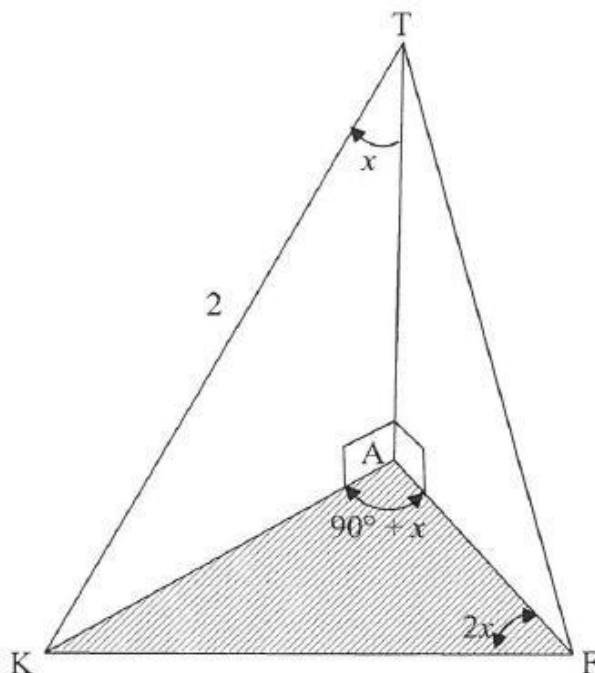
5.4 5.4.1 For which value(s) of x will $\frac{2 \tan x - \sin 2x}{2 \sin^2 x}$ be undefined in the interval $0^\circ \leq x \leq 180^\circ$? (3)

5.4.2 Prove the identity: $\frac{2 \tan x - \sin 2x}{2 \sin^2 x} = \tan x$ (6)

[26]

QUESTION 6

- 6.1 In the figure, points K, A and F lie in the same horizontal plane and TA represents a vertical tower. $\hat{ATK} = x$, $\hat{KAF} = 90^\circ + x$ and $\hat{KFA} = 2x$ where $0^\circ < x < 30^\circ$. $TK = 2$ units.



- 6.1.1 Express AK in terms of $\sin x$. (2)
- 6.1.2 Calculate the numerical value of KF. (5)

QUESTION/VRAAG 5

5.1	$x^2 + y^2$ $= (3 \sin \theta)^2 + (3 \cos \theta)^2$ $= 9 \sin^2 \theta + 9 \cos^2 \theta$ $= 9(\sin^2 \theta + \cos^2 \theta)$ $= 9(1)$ $= 9$	✓ simpl/vereen ✓ CF/GF = 9 ✓ answer/antw (3)
5.2	$\sin(540^\circ - x) \cdot \sin(-x) - \cos(180^\circ - x) \cdot \sin(90^\circ + x)$ $\sin(180^\circ - x) \cdot \sin(-x) - \cos(180^\circ - x) \cdot \sin(90^\circ + x)$ $= (\sin x)(-\sin x) - (-\cos x)(\cos x)$ $= -\sin^2 x + \cos^2 x$ $= \cos 2x$	✓ $\sin(540^\circ - x) = \sin x$ ✓ $\sin(-x) = -\sin x$ ✓ $\cos(180^\circ - x) = -\cos x$ ✓ $\sin(90^\circ + x) = \cos x$ ✓ $-\sin^2 x + \cos^2 x$ ✓ $\cos 2x$ (6)
5.3.1	$OT = \sqrt{x^2 + p^2}$ $\sin \alpha = \frac{y_T}{OT}$ $= \frac{p}{\sqrt{x^2 + p^2}}$ $\frac{p}{\sqrt{x^2 + p^2}} = \frac{p}{\sqrt{1 + p^2}}$ $x^2 = 1$ $x = -1$ OR/OF (P lies in 3rd quadrant) $x^2 + y^2 = r^2$ $x^2 + p^2 = (\sqrt{1 + p^2})^2$ $x^2 + p^2 = 1 + p^2$ $x^2 = 1$ $x = -1$ (P lies in 3rd quadrant)	✓ $OT = \sqrt{x^2 + p^2}$ ✓ $\sin \alpha = \frac{y_T}{OT}$ ✓ $x^2 = 1$ (3) ✓ $x^2 + y^2 = r^2$ ✓ subst ✓ $x^2 = 1$ (3)
5.3.2	$\cos(180^\circ + \alpha)$ $= -\cos \alpha$ $= -\left(\frac{-1}{\sqrt{1 + p^2}}\right)$ $= \frac{1}{\sqrt{1 + p^2}}$	✓ $-\cos \alpha$ ✓ answer/antw (2)

5.3.3	$\begin{aligned}\cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= \left(\frac{-1}{\sqrt{1+p^2}} \right)^2 - \left(\frac{p}{\sqrt{1+p^2}} \right)^2 \\ &= \frac{1}{1+p^2} - \frac{p^2}{1+p^2} \\ &= \frac{1-p^2}{1+p^2}\end{aligned}$ OR/OF $\begin{aligned}\cos 2\alpha &= 1 - 2\sin^2 \alpha \\ &= 1 - 2\left(\frac{p}{\sqrt{1+p^2}} \right)^2 \\ &= 1 - 2\left(\frac{p^2}{1+p^2} \right) \\ &= 1 - \frac{2p^2}{1+p^2} \\ &= \frac{1+p^2-2p^2}{1+p^2} \\ &= \frac{1-p^2}{1+p^2}\end{aligned}$ OR/OF $\begin{aligned}\cos 2\alpha &= 2\cos^2 \alpha - 1 \\ &= 2\left(\frac{-1}{\sqrt{1+p^2}} \right)^2 - 1 \\ &= 2\left(\frac{1}{1+p^2} \right) - 1 \\ &= \frac{2}{1+p^2} - 1 \\ &= \frac{2-1-p^2}{1+p^2} \\ &= \frac{1-p^2}{1+p^2}\end{aligned}$	✓ expansion/ uitbreiding ✓✓ squaring each term/kwadreer elke term (3) ✓ expansion/ uitbreiding ✓ squaring/kwadrering ✓ writing as single fraction/skryf as enkelterm (3) ✓ expansion/ uitbreiding ✓ squaring/kwadrering ✓ writing as single fraction/skryf as enkelterm (3)
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5.4.1	The identity is undefined for/<i>die identiteit is ongedefinieerd as:</i> $2\sin^2 x = 0$ $\therefore \sin x = 0; x = 0^\circ; 180^\circ$ or/of $\tan x = \infty; x = 90^\circ$ $\therefore x = 0^\circ; 90^\circ; 180^\circ$	✓ $x = 0^\circ$ ✓ $x = 90^\circ$ ✓ $x = 180^\circ$ (3)
5.4.2	LHS/LK = $\frac{2 \tan x - \sin 2x}{2 \sin^2 x}$ $= \frac{2 \left(\frac{\sin x}{\cos x} \right) - 2 \sin x \cos x}{2 \sin^2 x}$ $= \left(\frac{2 \sin x - 2 \sin x \cos^2 x}{\cos x} \right) \times \frac{1}{2 \sin^2 x}$ $= \frac{2 \sin x (1 - \cos^2 x)}{\cos x} \times \frac{1}{2 \sin^2 x}$ $= \frac{2 \sin x (\sin^2 x)}{\cos x} \times \frac{1}{2 \sin^2 x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ = RHS/RK OR/OF LHS/LK = $\frac{2 \tan x - \sin 2x}{2 \sin^2 x}$ $= \frac{2 \left(\frac{\sin x}{\cos x} \right) - 2 \sin x \cos x}{2 \sin^2 x} \times \frac{\cos x}{\cos x}$ $= \frac{2 \sin x - 2 \sin x \cos^2 x}{2 \sin^2 x \cos x}$ $= \frac{2 \sin x (1 - \cos^2 x)}{2 \sin^2 x \cos x}$ $= \frac{2 \sin x \cdot \sin^2 x}{2 \sin^2 x \cos x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ = RHS/RK	✓ $\frac{\sin x}{\cos x}$ ✓ $2 \sin x \cdot \cos x$ ✓ simplify numerator/ <i>vereenv teller</i> ✓ factorising/<i>fakt</i> ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplify to/<i>vereenv</i> na $\frac{\sin x}{\cos x}$ (6) ✓ $\frac{\sin x}{\cos x}$ ✓ $2 \sin x \cdot \cos x$ ✓ simpl/<i>vereenv</i> ✓ factorising/<i>fakt</i> ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplify to /<i>vereenv</i> na $\frac{\sin x}{\cos x}$ (6) [26]

6.1.1	In $\triangle TAK$: $\frac{AK}{KT} = \sin \hat{KTA}$ $AK = KT \cdot \sin x$ $= 2 \sin x$ OR/OF $\frac{\sin \hat{KTA}}{AK} = \frac{\sin \hat{KAT}}{KT}$ $\frac{\sin 90^\circ}{2} = \frac{\sin x}{AK}$ $AK = 2 \sin x$	✓ correct trig ratio/ korrekte trigverh. ✓ answer/antw (2) ✓ correct subst into sine rule/korrekte subst in sin-reël ✓ answer/antw (2)
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6.1.2	In $\triangle AKF$: $\frac{KF}{\sin \hat{KAF}} = \frac{AK}{\sin \hat{AFK}}$ $\frac{KF}{\sin(90^\circ + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK \cdot \sin(90^\circ + x)}{\sin 2x}$ $= \frac{2 \sin x \cdot \cos x}{2 \sin x \cdot \cos x}$ $= 1$ OR/OF In $\triangle AKF$: $\frac{KF}{\sin \hat{KAF}} = \frac{AK}{\sin \hat{AFK}}$ $\frac{KF}{\sin(90^\circ + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK \cdot \sin(90^\circ + x)}{\sin 2x}$ $= \frac{AT \cdot \tan x \cdot \cos x}{2 \sin x \cdot \cos x}$ $= \frac{2 \cos x \cdot \frac{\sin x}{\cos x} \cdot \cos x}{2 \sin x \cdot \cos x}$ $= 1$ $\cos x = \frac{AT}{2}$ $\therefore AT = 2 \cos x$	✓ using sine rule/ gebruik sin-reël ✓ correct subst into sine rule/korrekte subst in sin-reël ✓ $\sin(90^\circ + x) = \cos x$ ✓ $2 \sin x \cdot \cos x$ ✓ 1 (5) ✓ using sine rule/ gebruik sin-reël ✓ correct subst into sine rule/korrekte subst in sin-reël ✓ $\sin(90^\circ + x) = \cos x$ ✓ $2 \sin x \cdot \cos x$ ✓ 1 (5)
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QUESTION 5

5.1 Given that $\sin 23^\circ = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k , WITHOUT using a calculator:

5.1.1 $\sin 203^\circ$ (2)

5.1.2 $\cos 23^\circ$ (3)

5.1.3 $\tan(-23^\circ)$ (2)

5.2 Simplify the following expression to a single trigonometric function:

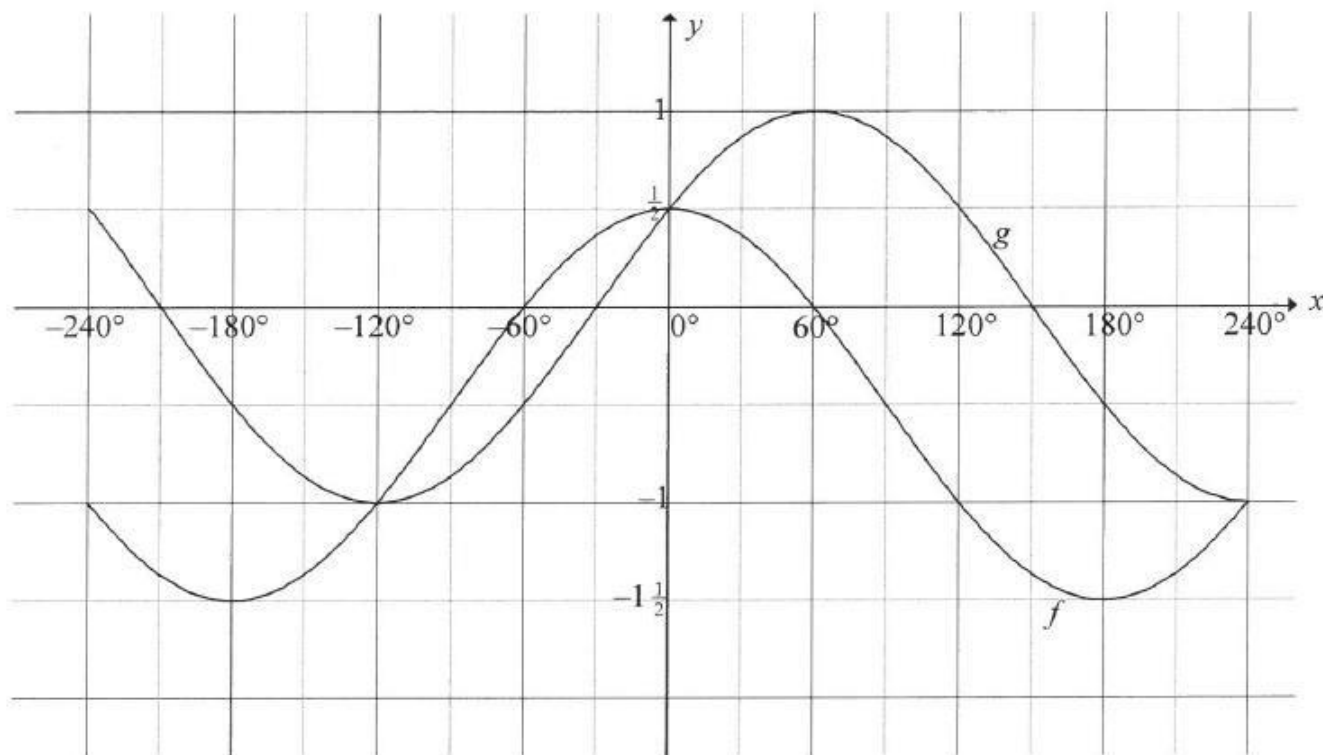
$$\frac{4 \cos(-x) \cdot \cos(90^\circ + x)}{\sin(30^\circ - x) \cdot \cos x + \cos(30^\circ - x) \cdot \sin x} \quad (6)$$

5.3 Determine the general solution of $\cos 2x - 7 \cos x - 3 = 0$. (6)

5.4 Given that $\sin \theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5)
[24]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240^\circ \leq x \leq 240^\circ$. The graphs intersect at $(0^\circ; \frac{1}{2})$, $(-120^\circ; -1)$ and $(240^\circ; -1)$.

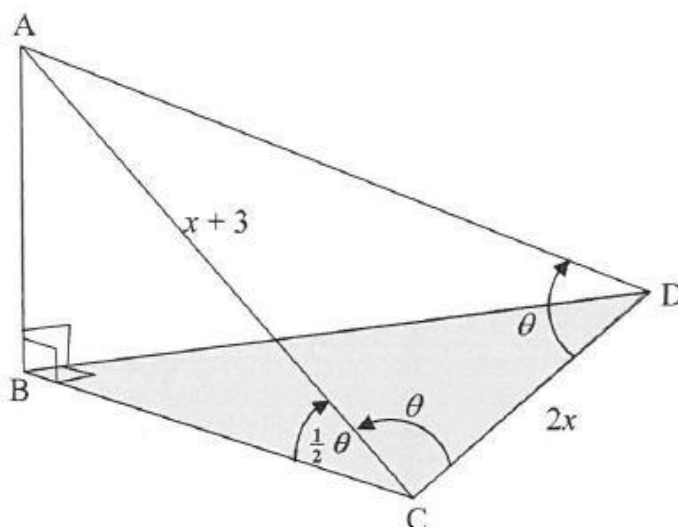


- 6.1 Determine the values of p and q . (4)
- 6.2 Determine the values of x in the interval $-240^\circ \leq x \leq 240^\circ$ for which $f(x) > g(x)$. (2)
- 6.3 Describe a transformation that the graph of g has to undergo to form the graph of h , where $h(x) = -\cos x$. (2)

[8]

QUESTION 7

A corner of a rectangular block of wood is cut off and shown in the diagram below. The inclined plane, that is, $\triangle ACD$, is an isosceles triangle having $\hat{ADC} = \hat{ACD} = \theta$. Also $\hat{ACB} = \frac{1}{2}\theta$, $AC = x + 3$ and $CD = 2x$.

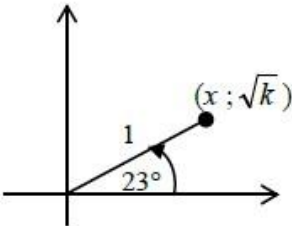


7.1 Determine an expression for $\hat{CAD}$ in terms of θ . (1)

7.2 Prove that $\cos \theta = \frac{x}{x+3}$. (4)

7.3 If it is given that $x = 2$, calculate AB , the height of the piece of wood. (5)
[10]

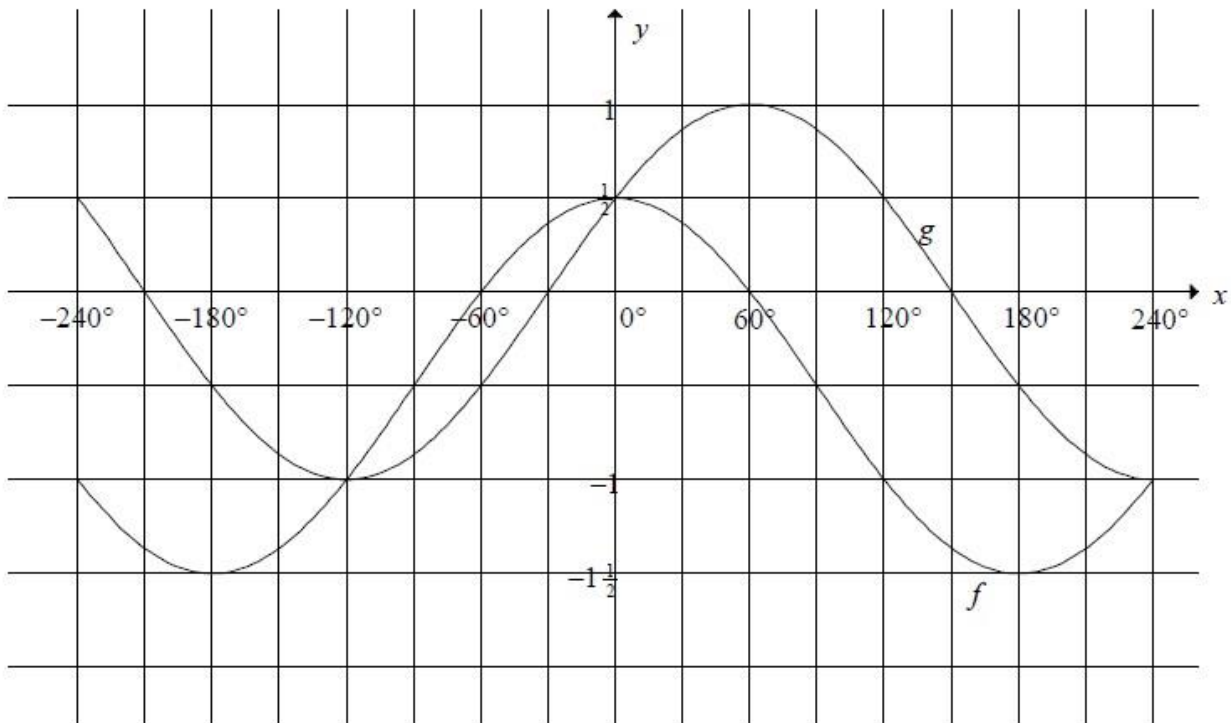
QUESTION/VRAAG 5

5.1.1	$\sin 203^\circ$ $= -\sin 23^\circ$ $= -\sqrt{k}$	✓ reduction/ reduksie ✓ answ ito/antw itv k (2)
5.1.2	$\cos^2 23^\circ = 1 - \sin^2 23^\circ$ $= 1 - k$ $\cos 23^\circ = \sqrt{1 - k}$ OR/OF $x^2 + (\sqrt{k})^2 = 1$ $x^2 = 1 - k$ $x = \sqrt{1 - k}$ $\cos 23^\circ = \frac{\sqrt{1 - k}}{1} = \sqrt{1 - k}$ 	✓ identity/identiteit ✓ $\cos^2 23^\circ$ ito/itv k ✓ answ/antw (3) ✓ $x^2 = 1 - k$ ✓ x ito/itv k ✓ answ/antw (3)
5.1.3	$\tan(-23^\circ) = -\tan 23^\circ$ $= -\frac{\sin 23^\circ}{\cos 23^\circ}$ $= -\frac{\sqrt{k}}{\sqrt{1 - k}} = -\sqrt{\frac{k}{1 - k}}$ OR/OF $\tan(-23^\circ) = -\tan 23^\circ$ $= -\frac{\sqrt{k}}{\sqrt{1 - k}} = -\sqrt{\frac{k}{1 - k}}$	✓ reduction/ reduksie ✓ answ ito/antw itv k (2) ✓ reduction/ reduksie ✓ answ ito/antw itv k (2)
5.2	$\frac{4 \cos x \cdot (-\sin x)}{\sin(30^\circ - x + x)}$ $= \frac{-4 \sin x \cdot \cos x}{\sin 30^\circ}$ $= \frac{-4 \sin x \cdot \cos x}{\frac{1}{2}}$ $= -8 \sin x \cdot \cos x$ $= -4(2 \sin x \cdot \cos x)$ $= -4 \sin 2x$	✓ $\cos x$ ✓ $-\sin x$ ✓ $\sin(\alpha + \beta)$ ✓ $\frac{1}{2}$ ✓ double sine form / dubbel sin form ✓ answ/antw (6)

OR/OF $\frac{4 \cos x \cdot (-\sin x)}{(\sin 30^\circ \cos x - \cos 30^\circ \sin x) \cos x + (\cos 30^\circ \cos x + \sin 30^\circ \sin x) \sin x}$ $= \frac{-4 \sin x \cdot \cos x}{\left(\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x\right) \cos x + \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x\right) \sin x}$ $= \frac{-2(2 \sin x \cdot \cos x)}{\frac{1}{2} \cos^2 x + \frac{1}{2} \sin^2 x}$ $= \frac{-2(2 \sin x \cdot \cos x)}{\frac{1}{2}(\cos^2 x + \sin^2 x)}$ $= \frac{-2(2 \sin x \cdot \cos x)}{\frac{1}{2}(1)}$ $= -8 \cos x \sin x$ $= -4(2 \sin x \cos x)$ $= -4 \sin 2x$	✓ $\cos x \quad \checkmark - \sin x$ ✓ $\frac{1}{2} \cos^2 x + \frac{1}{2} \sin^2 x$ ✓ $\frac{1}{2}$ ✓ double sine form / dubbel sin form ✓ answ/antw (6)
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5.3	$\cos 2x - 7 \cos x - 3 = 0$ $2 \cos^2 x - 1 - 7 \cos x - 3 = 0$ $2 \cos^2 x - 7 \cos x - 4 = 0$ $(2 \cos x + 1)(\cos x - 4) = 0$ $\therefore \cos x = -\frac{1}{2} \text{ or/of } \cos x = 4 \text{ (no solution)}$ $\therefore x = 120^\circ + n.360^\circ \text{ or/of } x = 240^\circ + n.360^\circ ; n \in \mathbb{Z}$ OR/OF $\therefore x = \pm 120^\circ + n.360^\circ ; n \in \mathbb{Z}$	$\checkmark$ expansion/ uitbreiding $\checkmark$ $2 \cos^2 x - 7 \cos x - 4 = 0$ $\checkmark$ factors/faktore $\checkmark \cos x = -\frac{1}{2}$ $\checkmark 120^\circ \text{ \& } 240^\circ$ $\checkmark + n.360^\circ$ OR/OF $\checkmark \pm 120^\circ$ $\checkmark + n.360^\circ$ (6)
5.4	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ $= 2 \sin \theta \cos \theta \cos \theta + (1 - 2 \sin^2 \theta) \sin \theta$ $= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$ $= 3\left(\frac{1}{3}\right) - 4\left(\frac{1}{3}\right)^3$ $= 1 - \frac{4}{27}$ $= \frac{23}{27}$	$\checkmark$ expansion of/ uitbreiding van $\sin(2\theta + \theta)$ $\checkmark$ expansions of $\sin 2\theta$ AND $\cos 2\theta$ $\checkmark 1 - \sin^2 \theta$ $\checkmark$ subst $\checkmark$ answ/antw (5) [24]

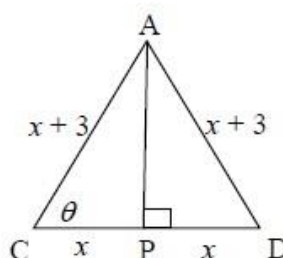
QUESTION/VRAG 6



6.1	$f(x) = \cos x - \frac{1}{2}$ and/en $g(x) = \sin(x + 30^\circ)$ $\therefore p = 30^\circ$ and/en $q = -\frac{1}{2}$ OR/OF $\sin(60^\circ + p) = 1$ and/en $\cos 0^\circ + q = \frac{1}{2}$ $\therefore p = 30^\circ$ $\therefore q = -\frac{1}{2}$	$\checkmark f(x) = \cos x - \frac{1}{2}$ $\checkmark g(x) = \sin(x + 30^\circ)$ $\checkmark$ value of/waarde v p $\checkmark$ value of/waarde v q (4) $\checkmark \sin(60^\circ + p) = 1$ $\checkmark \cos 0^\circ + q = \frac{1}{2}$ $\checkmark$ value of/waarde v p $\checkmark$ value of/waarde v q (4)
6.2	$x \in (-120^\circ ; 0^\circ)$ OR/OF $-120^\circ < x < 0^\circ$	$\checkmark$ critical values/ kritiese waardes $\checkmark$ correct interval/ korrekte interval (2)

6.3	The graph of g has to shift 60° to the left and then be reflected about the x-axis./Die grafiek van g moet 60° na links skuif en dan om die x-as gereflekteer word. OR/OF The graph of g must be reflected about the x-axis and then be shifted 60° to the left./Die grafiek van g moet om die x-as gereflekteer word en dan met 60° na links geskuif word. OR/OF The graph of g has to shift 120° to the right./Die grafiek van g moet 120° na regs geskuif word. OR/OF The graph of g has to shift 240° to the left./Die grafiek van g moet met 240° na links geskuif word	✓ 60° left/links ✓ reflection about x-axis/refleksie om x-as (2) ✓ reflection about x-axis/refleksie om x-as ✓ 60° left/links (2) ✓ ✓ 120° right/regs (2) ✓ ✓ 240° left/links (2) [8]
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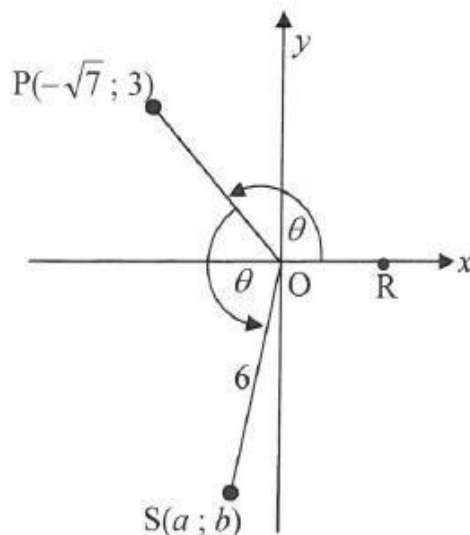
7.1	$\hat{CAD} = 180^\circ - 2\theta$ [$\angle$ s sum of Δ / $\angle$ e som van Δ]	✓ answ/antw (1)
7.2	$\frac{\sin \theta}{x+3} = \frac{\sin(180^\circ - 2\theta)}{2x}$ $\frac{\sin \theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin \theta}{x+3} = \frac{2 \sin \theta \cdot \cos \theta}{2x}$ $\cos \theta = \frac{2x \sin \theta}{2(x+3) \sin \theta}$ $\cos \theta = \frac{x}{x+3}$ OR/OF $AD = x + 3$ [sides opp = $\angle$s/sye to = $\angle$e] $AC^2 = AD^2 + CD^2 - 2AD \cdot CD \cdot \cos \theta$ $(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3) \cdot \cos \theta$ $0 = 4x^2 - 4x(x+3) \cos \theta$ $\cos \theta = \frac{4x^2}{4x(x+3)}$ $= \frac{x}{x+3}$ OR/OF Draw/Trek $AP \perp CD$ $\cos \theta = \frac{x}{x+3}$	✓ correct subst into sine rule/korrekte subst in sin-reël ✓ $\sin 2\theta$ ✓ $2 \sin \theta \cdot \cos \theta$ ✓ $\cos \theta$ as subject/as onderwerp (4) ✓ $AD = x + 3$ ✓ correct subst into cosine rule/korrekte subst in cos-reël ✓ simplification/vereenvoudiging ✓ $\cos \theta$ as subject/as onderwerp (4) ✓ ✓ constr/konstr ✓ ✓ sketch shown/toon skets (4)



7.3	$\cos \theta = \frac{2}{5}$ $\therefore \theta = 66,42^\circ$ In $\triangle ABC$: $\sin \frac{1}{2}\theta = \frac{AB}{AC}$ $\sin 33,21^\circ = \frac{AB}{5}$ $\therefore AB = 5 \sin 33,21^\circ$ $= 2,74$ OR/OF $\sin \frac{\theta}{2} = \frac{AB}{5}$ $\therefore AB = 5 \sin \frac{\theta}{2}$ but/maar: $\cos \theta = \frac{2}{5}$ $1 - 2 \sin^2 \frac{\theta}{2} = \frac{2}{5}$ $\sin^2 \frac{\theta}{2} = \frac{3}{10}$ $\sin \frac{\theta}{2} = \sqrt{\frac{3}{10}}$ $\therefore AB = 5 \sqrt{\frac{3}{10}} = \sqrt{\frac{15}{2}} = 2,74$	$\checkmark \cos \theta = \frac{2}{5}$ $\checkmark$ size of/grootte v θ $\checkmark$ correct ratio/ <i>korrekte verh</i> $\checkmark$ subst correctly/ <i>korrek</i> $\checkmark$ answ/antw (5) $\checkmark AB = 5 \sin \frac{\theta}{2}$ $\checkmark$ equation/vgl $\checkmark$ simplification/ <i>vereenvoudiging</i> $\checkmark$ value of/waarde v $\sin \frac{\theta}{2}$ $\checkmark$ answ/antw (5) [10]
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QUESTION 5

- 5.1 $P(-\sqrt{7}; 3)$ and $S(a; b)$ are points on the Cartesian plane, as shown in the diagram below. $\hat{POR} = \hat{POS} = \theta$ and $OS = 6$.



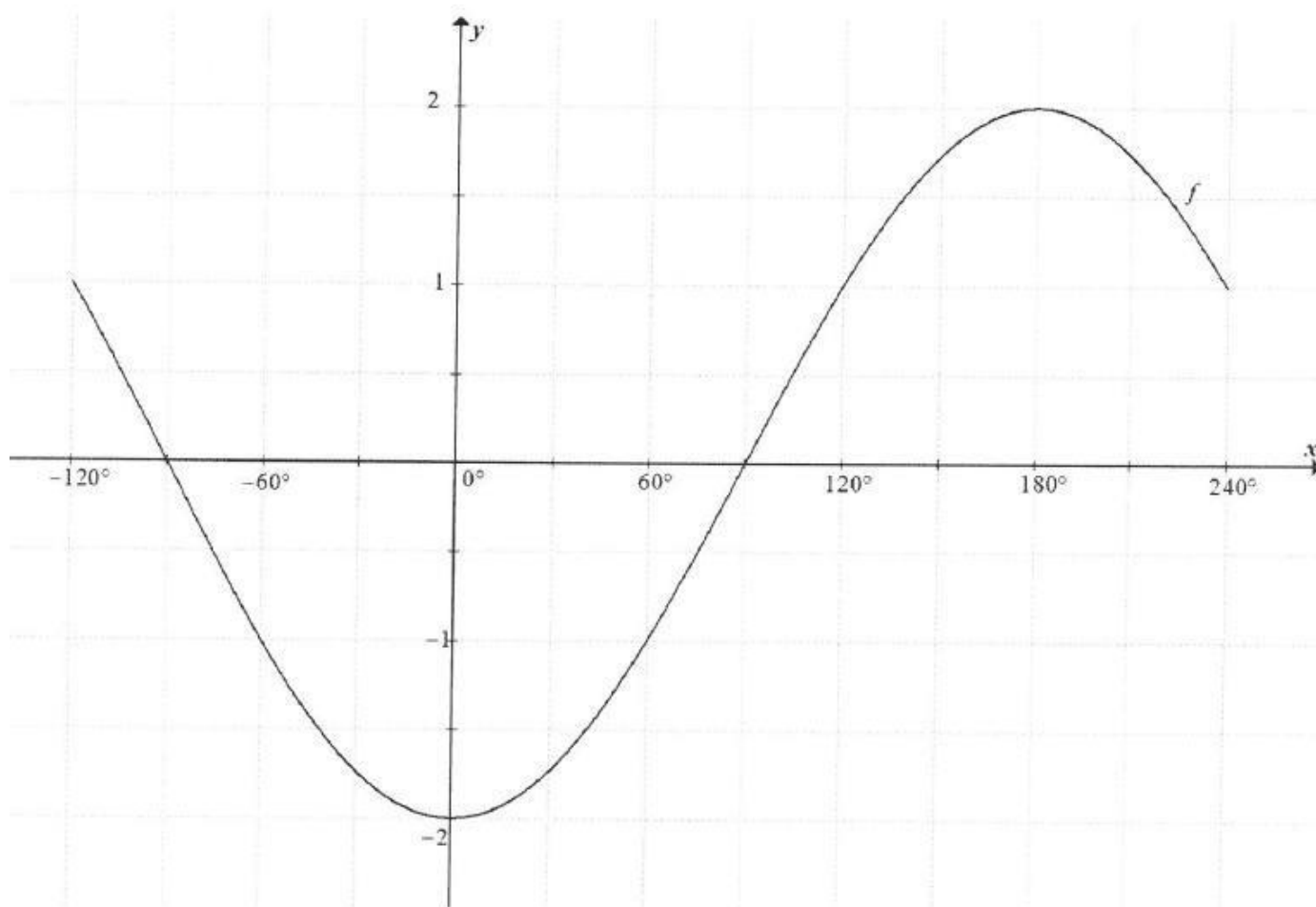
Determine, WITHOUT using a calculator, the value of:

- 5.1.1 $\tan \theta$ (1)
- 5.1.2 $\sin(-\theta)$ (3)
- 5.1.3 a (4)
- 5.2 5.2.1 Simplify $\frac{4 \sin x \cos x}{2 \sin^2 x - 1}$ to a single trigonometric ratio. (3)
- 5.2.2 Hence, calculate the value of $\frac{4 \sin 15^\circ \cos 15^\circ}{2 \sin^2 15^\circ - 1}$ WITHOUT using a calculator. (Leave your answer in simplest surd form.) (2)
- [13]

QUESTION 6

Given the equation: $\sin(x + 60^\circ) + 2\cos x = 0$

- 6.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)
- 6.2 Determine the solutions of the equation $\sin(x + 60^\circ) + 2\cos x = 0$ in the interval $-180^\circ \leq x \leq 180^\circ$. (3)
- 6.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120^\circ \leq x \leq 240^\circ$.

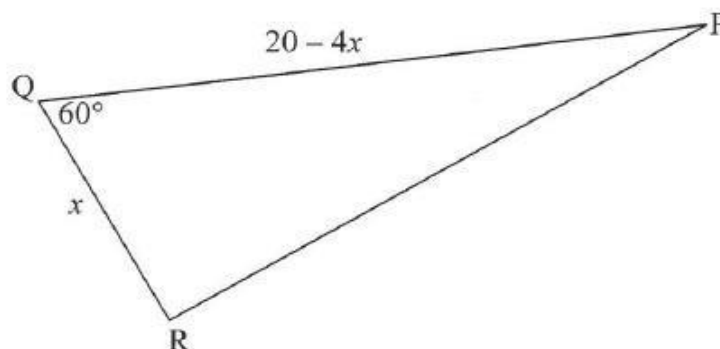


- 6.3.1 Draw the graph of $g(x) = \sin(x + 60^\circ)$ for $-120^\circ \leq x \leq 240^\circ$ on the grid provided in the ANSWER BOOK. (3)
- 6.3.2 Determine the values of x in the interval $-120^\circ \leq x \leq 240^\circ$ for which $\sin(x + 60^\circ) + 2\cos x > 0$. (3)

[13]

QUESTION 7

7.1 In the diagram below, $\triangle PQR$ is drawn with $PQ = 20 - 4x$, $RQ = x$ and $\hat{Q} = 60^\circ$.

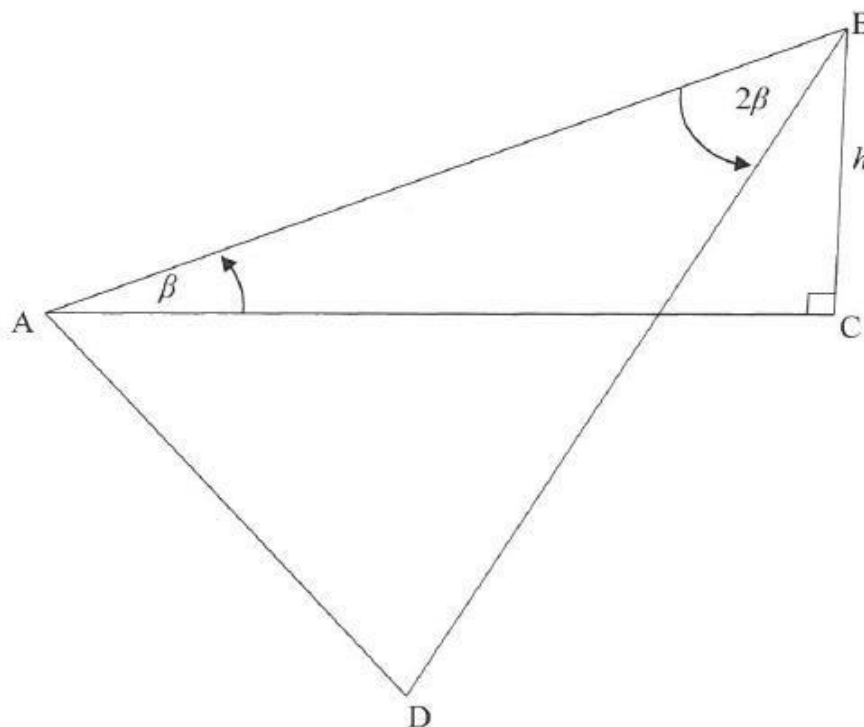


7.1.1 Show that the area of $\triangle PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

7.1.2 Determine the value of x for which the area of $\triangle PQR$ will be a maximum. (3)

7.1.3 Calculate the length of PR if the area of $\triangle PQR$ is a maximum. (3)

7.2 In the diagram below, BC is a pole anchored by two cables at A and D . A , D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole, B , is β . $\hat{ABD} = 2\beta$ and $BA = BD$.



Determine the distance AD between the two anchors in terms of h .

(7)
[15]

5.1.1	$\tan \theta = -\frac{3}{\sqrt{7}}$	✓ answ/antw (1)
5.1.2	$\sin(-\theta) = -\sin \theta$ $OP^2 = (-\sqrt{7})^2 + 3^2$ $OP^2 = 16$ $OP = 4$ $\sin(-\theta) = -\frac{3}{4}$	✓ reduction/ reduksie ✓ OP = 4 ✓ answ/antw (3)
5.1.3	$\frac{a}{6} = \cos 2\theta$ $a = 6(1 - 2\sin^2 \theta)$ $= 6 - 12\left(\frac{3}{4}\right)^2$ $= \frac{24}{4} - \frac{27}{4}$ $= -\frac{3}{4}$ OR/OF $\frac{a}{6} = \cos 2\theta$ $a = 6(2\cos^2 \theta - 1)$ $= 12\left(\frac{-\sqrt{7}}{4}\right)^2 - 6$ $= \frac{21}{4} - \frac{24}{4}$ $= -\frac{3}{4}$ OR/OF	✓ trig ratio/verh ✓ expansion/ uitbreiding ✓ $\sin \theta = \frac{3}{4}$ ✓ answ/antw (4) ✓ trig ratio/verh ✓ expansion/ uitbreiding ✓ $\cos \theta = \frac{-\sqrt{7}}{4}$ ✓ answ/antw (4)

	$\frac{a}{6} = \cos 2\theta$ $a = 6(\cos^2 \theta - \sin^2 \theta)$ $= 6 \left[\left(\frac{-\sqrt{7}}{4} \right)^2 - \left(\frac{3}{4} \right)^2 \right]$ $= 6 \left(-\frac{2}{16} \right)$ $= -\frac{3}{4}$	✓ trig ratio/verh ✓ expansion/ uitbreiding ✓ $\cos \theta = \frac{-\sqrt{7}}{4}$ & $\sin \theta = \frac{3}{4}$ ✓ answ/antw (4)
5.2.1	$\frac{4 \sin x \cdot \cos x}{2 \sin^2 x - 1} = \frac{2(2 \sin x \cdot \cos x)}{-(1 - 2 \sin^2 x)}$ $= \frac{2 \sin 2x}{-\cos 2x}$ $= -2 \tan 2x$	✓ $2 \sin 2x$ ✓ $-\cos 2x$ ✓ answ/antw (3)
5.2.2	$\frac{4 \sin 15^\circ \cos 15^\circ}{2 \sin^2 15^\circ - 1} = -2 \tan 2(15^\circ)$ $= -2 \tan 30^\circ$ $= -2 \left(\frac{1}{\sqrt{3}} \right)$ $= -\frac{2}{\sqrt{3}} \text{ OR/OR } -\frac{2\sqrt{3}}{3}$	✓ $-2 \tan 2(15^\circ)$ ✓ answ/antw (2) [13]

QUESTION/VRAAG 6

6.1	$\sin(x + 60^\circ) + 2\cos x = 0$ $\sin x \cos 60^\circ + \cos x \sin 60^\circ + 2\cos x = 0$ $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x + 2\cos x = 0$ $\frac{1}{2}\sin x = -2\cos x - \frac{\sqrt{3}}{2}\cos x$ $\sin x = -4\cos x - \sqrt{3}\cos x$ $\sin x = \cos x(-4 - \sqrt{3})$ $\frac{\sin x}{\cos x} = \frac{\cos x(-4 - \sqrt{3})}{\cos x}$ $\therefore \tan x = -4 - \sqrt{3}$	✓ expansion/uitbreiding ✓ special angle values/ spesiale $\angle$-waardes ✓ simpl/vereenv ✓ $\sin x = \cos x(-4 - \sqrt{3})$ (4)
6.2	$\tan x = -4 - \sqrt{3}$ $\tan x = -(4 + \sqrt{3})$ $\text{ref } \angle = 80,10^\circ$ $x = -80,1^\circ \text{ or/of } 99,9^\circ$	✓ $80,10^\circ$ ✓ $99,90^\circ$ ✓ $-80,1^\circ$ (3)
6.3.1		✓ $(30^\circ; 1)$ ✓ $(-60^\circ; 0)$ ✓ shape/vorm (3)
6.3.2	$\therefore \sin(x + 60^\circ) > -2\cos x$ $x \in (-80,10^\circ; 99,90^\circ) \text{ OR/OR } -80,10^\circ < x < 99,90^\circ$	✓✓ critical values/ kritiese waardes ✓ notation/notasie (3) [13]

QUESTION/VRAAG 7

7.1.1	Area of/Oppervlakte van $\Delta PQR = \frac{1}{2} PQ \cdot QR \cdot \sin \hat{Q}$ $= \frac{1}{2} x(20 - 4x)(\sin 60^\circ)$ $= 10x - 2x^2 \left(\frac{\sqrt{3}}{2} \right)$ $= 5\sqrt{3}x - \sqrt{3}x^2$	✓ subst into area rule/ subst in opp-reël ✓ subst & simpl/ subst en vereenv (2)
7.1.2	For maximum area/Vir maksimum opp: $(\text{Area } \Delta PQR)' = 0$ $5\sqrt{3} - 2\sqrt{3}x = 0$ $2\sqrt{3}x = 5\sqrt{3}$ $\therefore x_{\max} = \frac{5}{2} \text{ or } 2\frac{1}{2} \text{ or/of } 2,5$ OR/OF $x_{\max} = -\frac{b}{2a}$ $= -\frac{5\sqrt{3}}{2(-\sqrt{3})} = \frac{5}{2} \text{ or } 2\frac{1}{2} \text{ or } 2,5$ OR/OF $5\sqrt{3}x - \sqrt{3}x^2 = 0$ $\sqrt{3}x(5 - x) = 0$ $\therefore x = 0 \text{ or } 5$ $\therefore x_{\max} = \frac{0+5}{2} = \frac{5}{2} \text{ or/of } 2,5$	✓ $(\text{Area } \Delta PQR)' = 0$ ✓ $5\sqrt{3} - 2\sqrt{3}x$ ✓ answ/antw (3) ✓ formula/e ✓ subst ✓ answ/antw (3) ✓ x-intercepts/ x-afsnitte ✓ subst ✓ answ/antw (3)
7.1.3	$RP^2 = QP^2 + QR^2 - 2 \cdot QP \cdot QR \cdot \cos Q$ $= 10^2 + 2,5^2 - 2(10)(2,5) \cos 60^\circ$ $= 81,25$ $\therefore RP = 9,01$	✓ subst into cosine rule/in cos-reël ✓ simpl/vereenv ✓ answ/antw (3)

7.2	In $\triangle ABC$: $\sin \beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin \beta}$ In $\triangle ABD$: $AB = BD$ and/en $\hat{ADB} = 90^\circ - \beta$ [$\angle$s of/v $\triangle = 180^\circ$] $\frac{\sin 2\beta}{AD} = \frac{\sin(90^\circ - \beta)}{AB}$ $AD = \frac{AB \cdot \sin 2\beta}{\sin(90^\circ - \beta)}$ $= \frac{h}{\sin \beta} \times \frac{2 \sin \beta \cdot \cos \beta}{\cos \beta}$ $= 2h$ OR/OF In $\triangle ABC$: $\sin \beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin \beta}$ In $\triangle ABD$: $AB = BD$ $AD^2 = AB^2 + AB^2 - 2AB \cdot AB \cdot \cos 2\beta$ $= \left(\frac{h}{\sin \beta}\right)^2 + \left(\frac{h}{\sin \beta}\right)^2 - 2\left(\frac{h}{\sin \beta}\right)^2 \cdot \cos 2\beta$ $= \left(\frac{h}{\sin \beta}\right)^2 + \left(\frac{h}{\sin \beta}\right)^2 - 2\left(\frac{h}{\sin \beta}\right)^2 (1 - 2 \sin^2 \beta)$ $= \left(\frac{h}{\sin \beta}\right)^2 + \left(\frac{h}{\sin \beta}\right)^2 - 2\left(\frac{h}{\sin \beta}\right)^2 + 4h^2$ $= 4h^2$ $\therefore AD = 2h$ OR/OF Split isosceles triangle ABQ into two congruent triangles AEB and DEB. Then $\triangle ABC \equiv \triangle BAE$ ($AB = AC$, $\hat{ABE} = \hat{BAC} = \beta$, h) $\therefore AE = ED = BC = h$ $\therefore AD = 2h$	✓ AB ito h and/en β ✓ $\hat{ADB} = 90^\circ - \beta$ ✓ correct subst into cosine rule/subst korrek in cos-reël ✓ AD as subject/ onderwerp ✓ expansion/uitbrei $\sin(90^\circ - \beta)$ $= \cos \beta$ ✓ answer ito h (7) ✓ AB ito h and/en β ✓ correct subst into cosine rule/subst korrek in cos-reël ✓ expansion/uitbrei ✓ multiplication/ vermenigv ✓ simpl/vereenv ✓ answer ito h (7) (7)
		[15]

QUESTION 5

5.1 Given: $\sin 16^\circ = p$

Determine the following in terms of p , **without using a calculator**.

5.1.1 $\sin 196^\circ$ (2)

5.1.2 $\cos 16^\circ$ (2)

5.2 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Use the formula for $\cos(A - B)$ to derive a formula for $\sin(A + B)$ (3)

5.3 Simplify $\frac{\sqrt{1 - \cos^2 2A}}{\cos(-A) \cdot \cos(90^\circ + A)}$ completely, given that $0^\circ < A < 90^\circ$. (5)

5.4 Given: $\cos 2B = \frac{3}{5}$ and $0^\circ \leq B \leq 90^\circ$

Determine, **without using a calculator**, the value of EACH of the following in its simplest form:

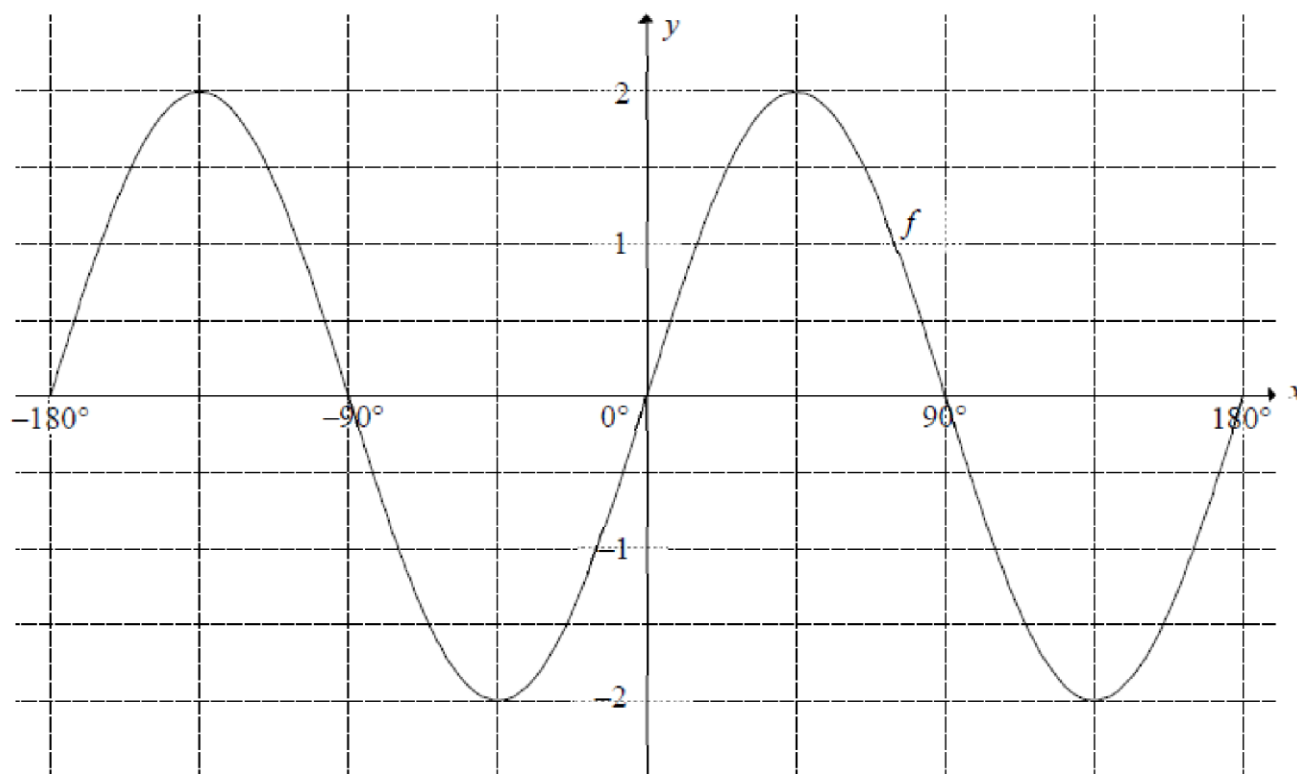
5.4.1 $\cos B$ (3)

5.4.2 $\sin B$ (2)

5.4.3 $\cos(B + 45^\circ)$ (4)
[21]

QUESTION 6

In the diagram the graph of $f(x) = 2 \sin 2x$ is drawn for the interval $x \in [-180^\circ; 180^\circ]$.

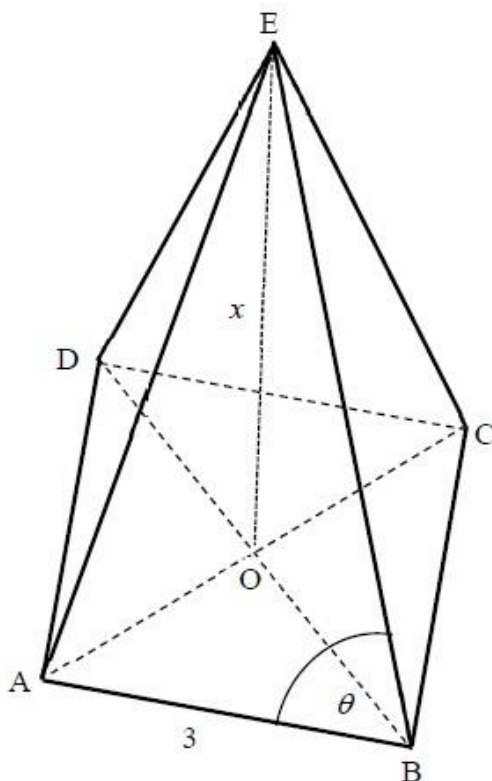


- 6.1 On the system of axes on which f is drawn in the ANSWER BOOK, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180^\circ; 180^\circ]$. Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)
 - 6.2 Write down the maximum value of $f(x) - 3$. (2)
 - 6.3 Determine the general solution of $f(x) = g(x)$. (4)
 - 6.4 Hence, determine the values of x for which $f(x) < g(x)$ in the interval $x \in [-180^\circ; 0^\circ]$. (3)
- [12]

QUESTION 7

E is the apex of a pyramid having a square base ABCD. O is the centre of the base. $\angle EBA = \theta$, $AB = 3$ m and EO, the perpendicular height of the pyramid, is x .

$$\text{Volume of pyramid} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$$



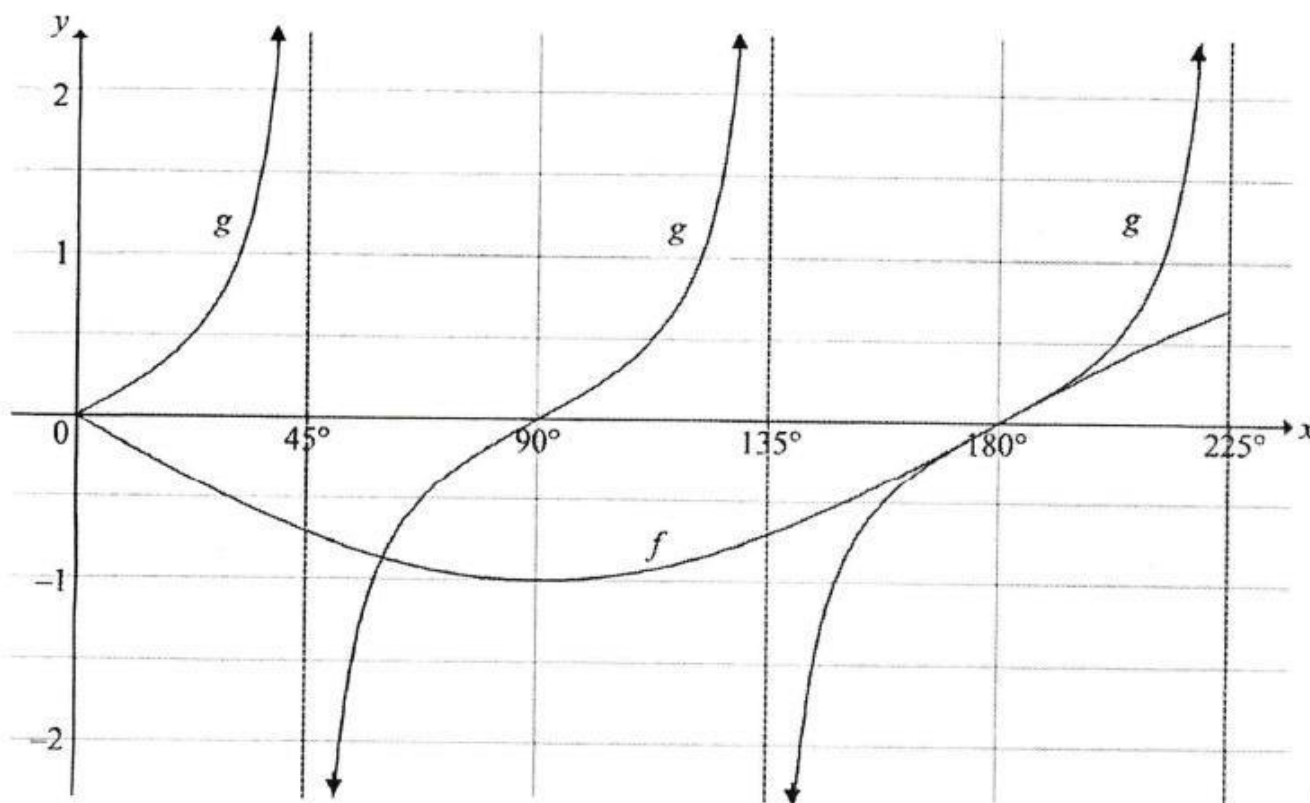
7.1 Calculate the length of OB. (3)

7.2 Show that $\cos \theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$ (5)

7.3 If the volume of the pyramid is 15 m^3 , calculate the value of θ . (4)
[12]

QUESTION 5

In the diagram, the graphs of the functions $f(x) = a \sin x$ and $g(x) = \tan bx$ are drawn on the same system of axes for the interval $0^\circ \leq x \leq 225^\circ$.



- 5.1 Write down the values of a and b . (2)
 - 5.2 Write down the period of $f(3x)$. (2)
 - 5.3 Determine the values of x in the interval $90^\circ \leq x \leq 225^\circ$ for which $f(x) \cdot g(x) \leq 0$. (3)
- [7]

QUESTION 6

6.1 Without using a calculator, determine the following in terms of $\sin 36^\circ$:

6.1.1 $\sin 324^\circ$ (1)

6.1.2 $\cos 72^\circ$ (2)

6.2 Prove the identity: $1 - \frac{\tan^2 \theta}{1 + \tan^2 \theta} = \cos^2 \theta$ (4)

6.3 Use QUESTION 6.2 to determine the general solution of:

$$1 - \frac{\tan^2 \frac{1}{2}x}{1 + \tan^2 \frac{1}{2}x} = \frac{1}{4}$$
 (6)

6.4 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

6.4.1 Use the formula for $\cos(A - B)$ to derive a formula for $\sin(A - B)$. (4)

6.4.2 Without using a calculator, show that

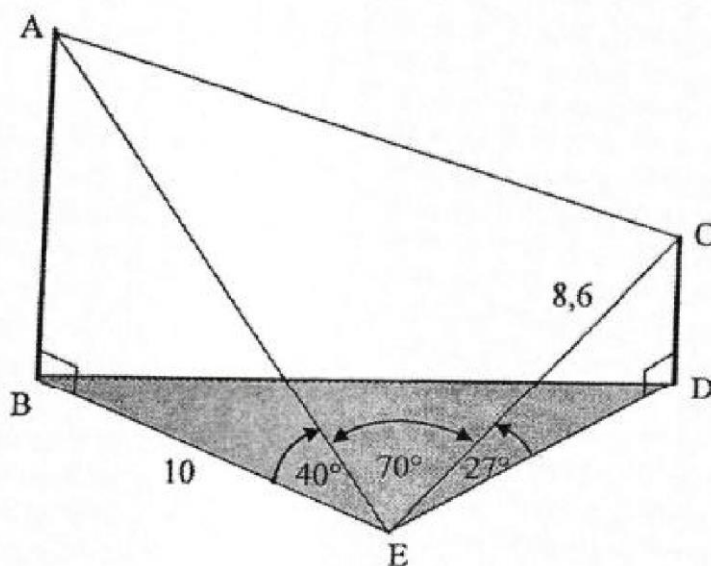
$$\sin(x + 64^\circ) \cos(x + 379^\circ) + \sin(x + 19^\circ) \cos(x + 244^\circ) = \frac{1}{\sqrt{2}}$$

for all values of x .

(6)
[23]

QUESTION 7

In the diagram, B, E and D are points in the same horizontal plane. AB and CD are vertical poles. Steel cables AE and CE anchor the poles at E. Another steel cable connects A and C. $CE = 8,6$ m, $BE = 10$ m, $\hat{AEB} = 40^\circ$, $\hat{AEC} = 70^\circ$ and $\hat{CED} = 27^\circ$.



Calculate the:

- | | | |
|-----|--------------------|-----|
| 7.1 | Height of pole CD | (2) |
| 7.2 | Length of cable AE | (2) |
| 7.3 | Length of cable AC | (4) |
| | | [8] |

<<<<<< THE END >>>>>>